10, 11, 12, 13 e 14 de novembro de 2025
ISSN 2177-3866

Comparative Analysis of Volatility Proxies and Regime-Based Asset Allocation

WANDERCI ALVES BITENCOURT
UNIVERSIDADE FEDERAL DE MINAS GERAIS (UFMG)

ROBERT ALDO IQUIAPAZA
UNIVERSIDADE FEDERAL DE MINAS GERAIS (UFMG)

Agradecimento à orgão de fomento:
We thank FAPEMIG and CNPq

Comparative Analysis of Volatility Proxies and Regime-Based Asset Allocation

1. Introduction

The growing vulnerability of global financial markets has exposed the limitations of traditional, static asset allocation strategies, such as the Mean-Variance model (Asgharian, Christiansen & Hou, 2023; Bekaert & Hoerova, 2014). These single-period approaches often fail to handle the abrupt changes in correlations, volatility, and downside risk characteristic of modern markets, compromising their ability to adapt (Ang & Timmermann, 2012; Kolm, Tütüncü & Fabozzi, 2014; MacLean, Yu & Zhao, 2022; Nystrup et al., 2019; Tu, 2010). This challenge is particularly acute in emerging economies, where market dynamics are more intense.

To address these shortcomings, regime-switching models like the Hidden Markov Model (HMM) have become essential tools, enabling the identification of latent market states with distinct statistical properties (Haghani & Dewey, 2016; Kolm, Tütüncü & Fabozzi, 2014; Nystrup et al., 2015; Zucchini & MacDonald, 2009). The effectiveness of an HMM is intrinsically linked to its input variables, and as volatility is a critical factor in risk management Bai and Cai (2024), the choice of its proxy is paramount. A well-chosen volatility measure allows for the modeling of distinct return distributions within each regime, which is fundamental for creating adaptive strategies that can optimize returns and minimize losses.

Despite extensive research on dynamic allocation using regime-switching models (Campani, Garcia & Lewin, 2021; Davoodi, Fereydooni & Rastegar, 2024; Kritzman, Page & Turkington, 2012; Nystrup, Madsen & Lindström, 2018; Nystrup et al., 2015, 2019; Oprisor & Kwon, 2021), a significant gap persists in the literature. As highlighted by Ardia, Bluteau and Rüede (2019), there is a lack of comparative analysis on the effectiveness of different volatility measures for identifying and forecasting market regimes via HMMs. This leads to this study's central research question: which volatility proxy best identifies market regimes, and how does this identification impact the efficiency of asset allocation strategies?

This study fills this gap by empirically comparing three volatility measures (historical, implied, and conditional) in the Brazilian and U.S.A. markets during the turbulent 2019-2024 period. This timeframe, marked by the COVID-19 pandemic and other major geopolitical and economic shocks, provides a robust testing ground for adaptive strategies. The most effective proxy is then used to construct a regime-based allocation strategy, which is evaluated against single-regime and naive (1/n) benchmarks. By doing so, this research provides unprecedented empirical evidence on the importance of the choice of volatility proxy and offers practical implications for investors seeking to optimize asset allocation in volatile environments.

2. Literature Review

The concepts of portfolio optimization and diversification have been fundamental to the development and understanding of financial markets, as well as to investment decision-making (Kolm, Tütüncü & Fabozzi, 2014). In general, investors seek to maximize returns while minimizing risks, balancing investment objectives with their respective risk profiles (Markowitz, 1952).

Modern Portfolio Theory, proposed by Markowitz (1952), provides a quantitative basis for portfolio management, using expected returns and asset covariances to establish a trade-off between risk and return. However, as it is a single-period model, when applied without proper adjustments to variable scenarios over multiple periods, it can be sensitive to estimation errors (MacLean, Zhao & Zhang, 2025). This sensitivity and the assumption of stable relationships over time can limit its effectiveness, especially in high volatility scenarios (Michaud, 1989; Tu, 2010). These challenges suggest the need for dynamic models capable of capturing changes in market conditions and adapting to them.

Regime-switching models have been widely explored to overcome these limitations, allowing portfolio selection strategies to incorporate the dynamics of financial markets (Kolm, Tütüncü & Fabozzi, 2014). In particular, models based on Markov Chains have gained prominence since the seminal work of Hamilton (1989), as they make it possible to map transitions between distinct economic states, such as recessions and expansions (Ang & Timmermann, 2012). This approach has been increasingly applied to model time series subject to regime changes (Ang & Timmermann, 2012; Zucchini & MacDonald, 2009).

Within this context, the HMM stands out for its remarkable ability to identify latent states in financial time series (Zucchini & MacDonald, 2009). This probabilistic model, which operates under the memorylessness assumption, models a time series as a Markov chain, allowing for the inference of unobservable regimes directly from measurable data, such as returns and volatility. Furthermore, as highlighted by Fereydooni and Mahootchi (2023), the segmentation of time series into upward and downward trends with the help of HMMs makes it possible to capture and incorporate the non-linearity inherent in the data.

The flexibility of the HMM makes it widely used in finance, with applications in dynamic asset allocation (Kritzman, Page & Turkington, 2012; Nguyen & Nguyen, 2015; Nystrup et al., 2019; Oprisor & Kwon, 2021), analysis of the stylized facts of returns (Nystrup et al., 2015), and identification of different market volatility clusters (Ardia et al., 2018). In particular, volatility forecasting is an essential challenge for portfolio management and hedging strategies, reinforcing the importance of sophisticated models for capturing risk regimes (Bai & Cai, 2024).

The HMM has proven to be especially effective in adapting to different market regimes, enabling the early detection of periods of economic stress and the implementation of more robust strategies (Bae, Kim & Mulvey, 2014; Fons et al., 2021; MacLean, Yu & Zhao, 2022; Nystrup, Madsen & Lindström, 2018; Peng, Kim & Mittnik, 2022; Uysal & Mulvey, 2021). This approach allows for an understanding of the relationship between market regimes and volatility, facilitating the identification of patterns that affect asset prices and improving decision-making.

The strong association between high volatility and financial crises underscores the need for sophisticated forecasting models that can identify market regimes (Chauvet, Senyuz & Yoldas, 2015; Megaritis, Vlastakis & Triantafyllou, 2021). While the GARCH model is standard for estimating conditional volatility, its limitations in capturing abrupt regime changes have led to the development of more advanced approaches. The literature shows that incorporating regime-switching into GARCH models significantly improves volatility modeling for various assets, from stock indices to cryptocurrencies (Ardia, Bluteau & Rüede, 2019; Ardia et al., 2018; Ma et al., 2019). Other forward-looking indicators, such as implied volatility, have also been shown to contain valuable information about future market regimes (Bekaert & Hoerova, 2014; Peng, Kim & Mittnik, 2022).

The ability to model market regimes has direct implications for portfolio allocation, as dynamic strategies that adapt to changing conditions have been shown to deliver superior performance. For example, MacLean, Zhao and Zhang (2025) demonstrate that making mean-variance optimization parameters regime-dependent leads to superior wealth growth compared to static approaches. However, despite the effectiveness of HMMs in this area, the choice of the observable variable used to estimate the latent states remains an under-investigated issue. While some works highlight the importance of conditional volatility Peng, Kim and Mittnik (2022) and others implied volatility Bekaert and Hoerova (2014), the literature lacks a direct empirical comparison of their effectiveness as proxies. This study fills this critical gap by evaluating the predictive ability of these distinct volatility measures for identifying market regimes in both developed and emerging markets.

Table 1: Description of finance metrics being analyzed.

Variable	Market	Description
Exchange-Traded Funds (ETFs):		
BOVA11	BR	Tracks the performance of the Ibovespa index.
SMAL11	BR	Tracks the performance of the Brazilian Small Caps index.
IMAB11	BR	Replicates Brazilian inflation-linked government bonds.
FIXA11	BR	Composed of Brazilian fixed-rate government bonds.
IVV	USA	Tracks the performance of the S&P500 index.
IWM	USA	Replicates U.S.A. Small Caps stocks - Russell 2000 index.
IEF	USA	Replicates medium-term fixed-rate U.S.A. Treasury bonds.
TIP	USA	Composed of U.S.A. Treasury Inflation-Protected Securities.
Risk-Free Rate:		
Selic	BR	Short-term interest rate set by the Central Bank of Brazil.
IRX	USA	Interest rate of the 13-week U.S.A. Treasury Bills (T-bills).

Note: BR refers to data from the Brazilian market (in BRL); USA refers to data from the U.S.A. market (in USD). All data are used daily. Returns are computed as log-returns from closing prices. The S&P 500, Ibovespa, ETFs, VIX, and IRX series were obtained from Yahoo Finance, and the Selic rate was retrieved from the Central Bank of Brazil.

3. Methodology

The following section presents the data and methodological procedures employed in this study, organized into two main stages. In the first stage, the implementation of the HMM is detailed, using different volatility proxies and rolling windows. The second stage presents the optimal asset allocation conditioned on the identified regimes, with its performance evaluated on out-of-sample data.

3. Data

To calculate the volatility measures, this study uses daily returns from the Ibovespa (Brazil) and the S&P 500 (U.S.A.) for the period from January 2019 to December 2024. This timeframe was chosen to capture the unprecedented financial instability of recent years (including the COVID-19 pandemic and subsequent macroeconomic shocks) and to align with the data availability of recently launched Brazilian fixed-income ETFs, thus ensuring a consistent analysis horizon for both markets.

Three volatility proxies were considered: historical, conditional, and implied. Historical volatility was calculated as the 21-day moving standard deviation of daily log-returns. Conditional volatility was estimated using a GARCH(1,1) model via maximum likelihood, implemented in the `rugarch` R package.

For implied volatility, the VIX index was used for the U.S.A. market. For the Brazilian market, a composite series was constructed from the IVol-BR (NEFIN, 2017) and VXBR (S&P Dow Jones Indices, 2024) indices due to the lack of a single continuous series. To further validate the use of a composite series for Brazilian implied volatility, we conducted statistical tests on the overlapping period (May 6, 2021, to April 29, 2022). A two-sample t-test showed no statistically significant difference between the means of the IVol-BR and VXBR series, and a Levene's test confirmed the homogeneity of their variances. These results, combined with the positive correlation of 67.4%, support the feasibility of the composite series to represent a single, consistent measure of implied volatility for the Brazilian market. All annualized indices were converted to a daily basis using the transformation vol daily $\text{vol}_{\text{daily}} = \text{vol}_{\text{annual}}/\sqrt{252}$.

In the asset allocation stage, four representative ETFs were used for each market, denominated in their respective local currencies without currency conversion. Table 1 presents a detailed description of these assets for the period from January 2020 to December 2024. , ensuring frequency standardization across all volatility measures.

The choice of exchange-traded funds (ETFs) for the asset allocation strategy is justified by their efficiency, accessibility, and trading flexibility (Matos, Iquiapaza & Ferreira, 2014). For this study, the selection aimed to create a diversified portfolio with both aggressive (equity) and conservative (fixed-income) assets, reflecting common investment strategies like the 60/40 allocation (Grant, Kwon & Satchell, 2024; Kandhari, 2024). In the Brazilian context specifically, ETFs represent a growing segment whose characteristics of diversification, security, and ease of access align well with the typical local investor profile (ANBIMA, 2024).

3. Hidden Markov Model - HMM

The HMM allows the modeling of phenomena by stochastic processes that describe the dynamics of observable events, Y_t (such as price movements), that are influenced by hidden states, X_t (such as volatility regimes or phases of the business cycle).

In this study, following Nystrup et al. (2019), we assume there are no gains in assuming more than two states, prioritizing the interpretability of the model's results and avoiding the risk of overfitting the data. Thus, in the context of this study, the model classifies volatility regimes as high or low, inferring their probabilities from historical data. In each regime, a Gaussian distribution is assumed, whose parameters depend on the underlying regime.

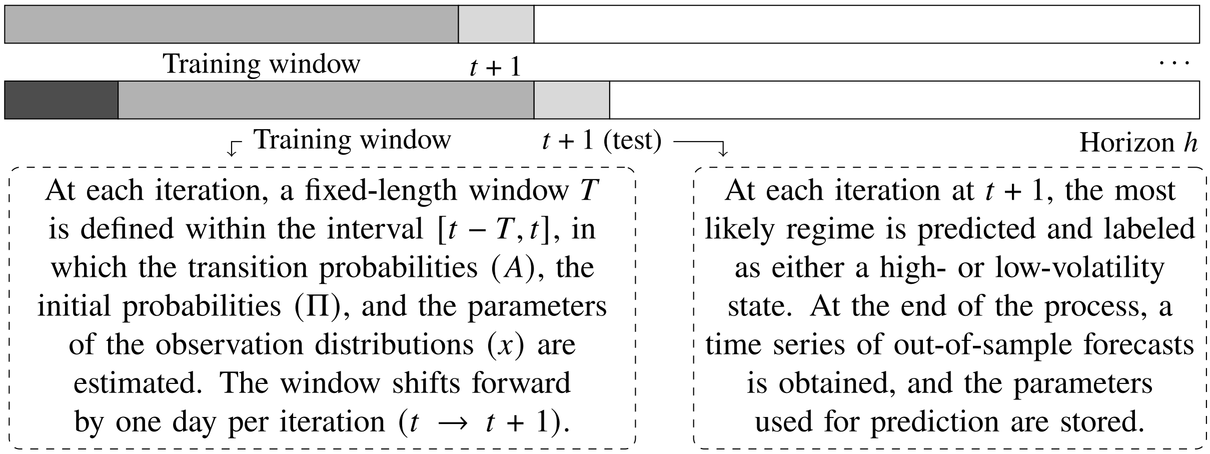
The HMM follows the first-order assumption of a Markov Chain, where each observation depends only on the state that produced it, being completely independent of any other state in the chain. Thus, the transitions between these two regimes are described by a transition matrix $A = \begin{bmatrix} a_{11} & 1 - a_{11} \\ 1 - a_{22} & a_{22} \end{bmatrix}$, where a_{11} represents the probability of remaining in the low volatility regime in the next period, given that the previous period was also of low volatility. Analogously, a_{22} corresponds to the probability of remaining in the high volatility regime, conditional on the fact that the previous period was already in that regime.

Considering two states, the forecast of future returns and the covariance matrices are based on the estimated parameters of the HMM. It is assumed that returns follow a normal distribution $N(\mu_1, \Sigma_1)$ in regime 1 and $N(\mu_2, \Sigma_2)$ in regime 2. The probability associated with the market state at time t is represented by q_t, then:

- i) the probability $\hat{q}t + 1$ that the market is under a low volatility regime at time $(t + 1)$ is: $\hat{q}_{t+1} = q_t a_{11} + (1 - q_t)(1 - a_{22})$;
- ii) the forecast of the expected return $\hat{\mu}t + 1$ at time $(t+1)$ is: $\hat{\mu}_{t+1} = \hat{q}_{t+1}\mu_1 + (1 - \hat{q}_{t+1})\mu_2$;
- iii) the forecast of the return covariance matrix $\hat{\Sigma}t + 1$ at time $(t + 1)$ is: $\hat{\Sigma}_{t+1} = (\hat{q}_{t+1}\Sigma_1 + (1 - \hat{q}_{t+1})\Sigma_2) + (\hat{q}_{t+1}(\mu_1 - \hat{\mu}_{t+1})^2 + (1 - \hat{q}_{t+1})(\mu_2 - \hat{\mu}_{t+1})^2)$.

The HMM parameters were estimated using the Expectation-Maximization and Viterbi algorithms, utilizing the depmixS4 package in the R software. For the estimation and evaluation of the model, the different volatility measures presented in Table 1 and moving windows of 126, 252, and 504 days were used. It is important to consider and test different moving window sizes so that (i) the estimation is based on the most recent market data and (ii) the data includes at least one complete volatility cycle. In their analysis of asset correlations over time, Jacobs and Karagozoglu (2014) identified that they vary significantly during financial crises, with some increasing substantially depending on market conditions. After testing various window sizes, they concluded that the ideal window for this type of analysis is between 190 and 240 trading days.

To ensure that the empirical analyses are conducted with out-of-sample test data, the estimation of the HMM parameters was carried out based on volatility measures over a moving window. The regime is forecasted one step ahead; that is, considering a data window of a fixed size of T



At each iteration, a fixed-length window T is defined within the interval $[t - T, t]$, in which the transition probabilities (A), the initial probabilities (Π), and the parameters of the observation distributions (x) are estimated. The window shifts forward by one day per iteration $(t \rightarrow t + 1)$.

At each iteration at $t + 1$, the most likely regime is predicted and labeled as either a high- or low-volatility state. At the end of the process, a time series of out-of-sample forecasts is obtained, and the parameters used for prediction are stored.

Figure 1: Rolling window representation for estimating the HMM.

days, the regime projection corresponds to day $t + 1$, with the first T days of the sample being used as a warm-up period. The implementation of this moving window approach is illustrated in Figure 1.

The labeling of regimes was performed using a cutoff of 0.5; that is, if the probability estimated by the HMM is greater than or equal to 0.5, the regime is classified as 1; otherwise, as 2. As the regime is forecasted at each instant t, over the sample horizon h, it is necessary to standardize the meaning of regimes 1 and 2 in terms of this study's central object, volatility. Thus, the regimes are classified as high or low volatility based on the estimated variability for each: the regime with lower variability is classified as a low-volatility regime, while the one with higher variability is identified as a high-volatility regime.

3.2.1 Evaluation Criterion for Volatility Proxies

The choice of the most suitable volatility proxy for identifying market regimes was based on comparing the predictive performance of the HMM models fitted to each proxy. For this, the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) were used, calculated out-of-sample. These metrics quantify the ability to forecast future regimes, providing an objective criterion for selecting the volatility measure that best captures the latent market dynamics. Thus, the proxy that presented the lowest RMSE and MAE in the out-of-sample predictions was used in the asset allocation optimization stage.

The MAE measures the average of the absolute differences between the predicted and observed values. It provides a direct estimate of the average magnitude of the errors:

$$\text{MAE} = 1/n \cdot \Sigma_{i=1}^{n} |y_i - \hat{y}_i| \tag{1}$$

The RMSE measures the square root of the average of the squared errors and penalizes larger errors more heavily:

$$\text{RMSE} = \sqrt{1/n \cdot \Sigma_{i=1}^{n} (y_i - \hat{y}_i)^2} \tag{2}$$

Where: y_i represents the observed value and $\hat{y}_i$ the predicted value, for n observations.

The choice of the estimation window was also based on comparing the out-of-sample RMSE and MAE, considering the predictive capacity of the HMM models.

3. Portfolios

To understand the optimal allocation in different volatility regimes, daily portfolios were adjusted based on the regime identified from the results obtained in the HMM fitting stage.

At each period t, with $t = 1, \ldots, h$, the optimization model is solved using information about the risk and return of the n assets available for allocation. These parameters are calculated based on the period of the moving window used for forecasting the regime in the current period and are conditioned on it.

The asset allocation strategy conditioned on the identified regimes was structured as a quadratic optimization problem, aiming to maximize the investor's expected utility while simultaneously penalizing risk and transaction costs, in a Long-Only strategy. The objective function adopted follows the weighted quadratic utility structure, as proposed by Boyd et al. (2017). An l_1 penalty function was adopted for turnover, or changes in weights relative to the previous period, according to Equation 3.

$$\begin{aligned} \max_{\mathbf{w}_t} \quad & E[\mathbf{r}_t^T \mathbf{w}_t] - (\lambda/2)\mathbf{w}_t^T \mathbf{\Sigma} \mathbf{w}_t - \gamma \|\mathbf{w}_t - \mathbf{w}_{t-1}\|_1 \\ & \text{s.t.} \quad \mathbf{w} \geq 0; \quad \mathbf{1}^T \mathbf{w} = 1 \end{aligned} \tag{3}$$

Where r_t is the vector of expected returns at time t; Σ represents the covariance matrix of returns; w_t is the vector of portfolio weights in period t; $\lambda \in [0, \infty)$ is the risk aversion parameter; and $\gamma \in [0, \infty)$ is the transaction cost parameter, which regulates the impact of changes in portfolio weights over time. The constraint $w_t \geq 0$ imposes that only long positions are permitted. The product $1^T w_t$ represents the sum of the weights, and being equal to 1 ensures that all available capital is fully allocated.

The optimization model described in Equation 3 was applied using the HMM setup identified as most suitable, with data beginning in January 2020 after the model's warm-up period. The model was tested with risk aversion parameters (λ) of 1, 5, and 10, and proportional transaction costs of 0.03%, 0.05%, and 0.1%. These parameter ranges are informed by the literature on rebalancing sensitivity and the need for calibration (Han, 2020; Lewin & Campani, 2024; Li, Uysal & Mulvey, 2022). The allocation was performed on four representative ETFs for each market (as described in Table 1), chosen for their diversification and liquidity benefits.

To evaluate the effectiveness of the proposed volatility regime-based strategy, its performance was compared against two benchmarks: a single-regime portfolio and a naive strategy. The single-regime portfolio follows the same optimization as the proposed model but ignores regime changes and performs quarterly rebalancing. The naive strategy consists of allocating capital equally among the assets ($w_i = 1/n$), serving as a robust benchmark that is often difficult to outperform Han (2020). The comparison between strategies was conducted using classic performance metrics: average return, risk (standard deviation of portfolio returns), the Sharpe ratio, and maximum drawdown.

4. Empirical Results

Table 2 presents the descriptive statistics of the series used as volatility proxies, employed in fitting the HMM parameters for the Brazilian and U.S.A. markets. It is observed that all measures exhibit positive skewness and high kurtosis, indicating asymmetric distributions with heavy tails—typical characteristics of financial time series. These features highlight the suitability of regime-switching models, such as the HMM, for handling these series, as traditional models may fail to adequately capture such non-linear and extreme aspects of financial data.

In the Brazilian market, implied volatility (VIXBR) shows a mean of 1.540 and volatility of 0.545, values higher than those presented by the historical and conditional volatility measures

Table 2: Descriptive statistics of the volatility proxy series.

	Series	Mean	Std. Dev.	Skewness	Kurtosis
BR	Implied Volatility (VIXBR)	1.540	0.545	4.08	34.46
	Historical Volatility (Ibovespa)	0.013	0.010	5.00	32.11
	Conditional Volatility (Ibovespa)	0.014	0.008	5.03	32.34
USA	Implied Volatility (VIX)	1.349	0.521	2.52	14.13
	Historical Volatility (S&P500)	0.011	0.008	3.88	22.54
	Conditional Volatility (S&P500)	0.011	0.008	4.26	28.07

Note: Volatility series from January 2020 to December 2024 with daily frequency.

of the Ibovespa, which have similar mean values. The same pattern is observed in the U.S.A., where implied volatility (VIX) exhibits a mean of 1.349 and a standard deviation of 0.521. The historical and conditional volatilities of the S&P 500, despite having identical means, show higher levels of skewness and kurtosis compared to implied volatility.

These results indicate that implied volatility, reflecting future expectations about risk, tends to be higher due to the uncertainty anticipated by investors. On the other hand, historical and conditional volatility more directly capture the recent dynamics of the market, which may explain their similarities. All analyzed proxies reveal non-normal statistical characteristics, reinforcing the importance of using flexible models, such as HMM, capable of adequately handling structural changes and extreme events present in financial series.

Following the proposed methodological procedures, we first discuss the results regarding the estimates obtained by the HMM. The performance results for the different volatility measures in identifying market regimes are presented in Table 3.

Table 3: A comparison of volatility proxies for HMM regime identification.

	Brazil			**United States**		
Window	**Implied** (RMSE / MAE)	**Conditional** (RMSE / MAE)	**Historical** (RMSE / MAE)	**Implied** (RMSE / MAE)	**Conditional** (RMSE / MAE)	**Historical** (RMSE / MAE)
126	0.369 / 0.163	0.004 / 0.001	0.005 / 0.002	0.322 / 0.185	0.005 / 0.002	0.004 / 0.002
252	0.408 / 0.205	0.005 / 0.002	0.006 / 0.003	0.383 / 0.242	0.006 / 0.003	0.006 / 0.003
504	0.423 / 0.212	0.006 / 0.002	0.007 / 0.003	0.425 / 0.249	0.006 / 0.003	0.006 / 0.003

Note: The Kruskal-Wallis test suggests statistically significant differences between volatility measures (RMSE: p-value = 0.03; MAE: p-value = 0.02), but not between the window sizes (RMSE: p-value = 0.13; MAE: p-value = 0.22).

Thus, conditional volatility can provide a more stable and consistent description of the market's volatility structure. GARCH models are recognized for capturing fundamental characteristics of financial series, such as volatility clustering and the asymmetry of informational shocks, advantages that are maintained even in regime-switching contexts (Ardia et al., 2018). Such characteristics are essential for distinguishing market regimes, which are possibly not as clearly reflected in implied volatility, as it is more focused on future expectations.

It should be noted that implied volatility, despite being a valuable forward-looking measure derived from option prices, can be influenced by factors external to the underlying asset's regime, such as general market sentiment, supply and demand for options, and specific risk premiums in the derivatives market. In contrast, conditional volatility more directly reflects the historical dynamics of the asset's or index's own returns (Aigner, 2022; Bekaert & Hoerova, 2014).

Next, we explore the segmentation of market regimes based on this proxy and evaluate its impact on portfolio allocation, considering a 252-day moving window (approximately one year). As

Table 4: ETF performance indicators by market and year.

	Brazil				U.S.A.			
Year	**BOVA**	**SMAL**	**IMAB**	**FIXA**	**IVV**	**IWM**	**IEF**	**TIP**
Annualized Excess Returns (%)								
2020	-8.64	-13.19	3.96	5.75	15.72	17.92	8.45	9.13
2021	-19.34	-25.12	-11.41	-13.36	27.01	13.42	-4.16	1.17
2022	-0.10	-22.81	-8.08	-5.64	-21.51	-23.69	-18.76	-19.67
2023	18.25	10.43	3.28	4.52	19.47	10.19	-4.40	-4.05
2024	-19.13	-32.00	-15.41	-15.46	18.86	5.03	-8.88	-5.71
Performance over the full period (annualized)								
Excess Return (%)	-2.95	-13.76	-1.63	-1.04	10.26	1.92	-3.74	-2.04
Volatility (%)	26.18	32.28	10.01	7.32	21.45	27.55	7.97	7.34
Sharpe Ratio	-0.11	-0.43	-0.16	-0.14	0.48	0.07	-0.47	-0.28
Adj. Sharpe Ratio	-0.008	-0.044	-0.002	-0.001	0.478	0.070	-0.003	-0.001

Note: Excess returns are calculated by subtracting the average annual risk-free rate from the average annual ETF return. The Selic (Brazil) and IRX (U.S.A.) serve as proxies for these rates. The Sharpe Ratio is the ratio between excess annual return and its volatility. The Adjusted Sharpe Ratio follows the adjustment by Israelsen (2005) when the excess return is negative.

discussed, the choice of the window did not show a statistically significant difference, thus becoming an arbitrary decision among the evaluated periods. However, the choice of 252 days appears to be the most reasonable, as it balances the need to capture regime dynamics without compromising the robustness of the analysis. This value is also close to the ideal reported by Jacobs and Karagozoglu (2014).

A 126-day window (approximately six months) could result in a reduced dataset for each regime, harming the allocation stage. On the other hand, a 504-day window (approximately two years) would increase the risk of deteriorating the volatility's predictive ability, as longer windows can excessively smooth the transitions between regimes. Thus, the choice of 252 days represents a suitable option between stability and predictive capacity.

Therefore, considering conditional volatility, a 252-day window, and the presence of two regimes, it was found that the frequency of transitions between regimes was similar in the analyzed markets. In the studied period, the Brazilian market switched between regimes 33 times (2.65%), while the U.S.A. market recorded 35 changes (2.78%). In terms of duration, Brazil remained in a high-volatility regime for approximately 30% of the analyzed period, whereas the United States spent 39% of the time in this regime.

The identified regimes align with established findings in financial literature. As expected, periods of low volatility were longer and more frequent than periods of high volatility, which are typically associated with crises. Furthermore, the results confirmed the stylized fact of volatility clustering, where the high persistence of regimes leads to prolonged periods of stability or turbulence rather than random occurrences (Asgharian, Christiansen & Hou, 2023; Chauvet, Senyuz & Yoldas, 2015; Li, Uysal & Mulvey, 2022; Ma et al., 2019).

A key counter-intuitive finding was that the U.S.A. market experienced a more prolonged high-volatility period than the Brazilian market. While emerging markets are typically more sensitive to external shocks (De Paula, Fritz & Prates, 2024), this result can be justified by the direct impact of recent geopolitical conflicts (the war in Ukraine and tensions in the Middle East) on the U.S.A. As economic and political uncertainty is a primary driver of volatility persistence (Asgharian, Christiansen & Hou, 2023; Douglas, Conrad & Frederick, 2025), these events likely had a more sustained impact on the U.S.A. market during the analyzed period.

The analysis of Table 4 reveals distinct asset behaviors shaped by a challenging post-COVID-19 macroeconomic environment. The sharp rise in global interest rates from 2022 onwards—fueled by inflation and geopolitical conflicts—increased the appeal of fixed-income securities, which consistently showed lower volatility than their equity counterparts. This context explains the lack of a significant risk premium observed in the Brazilian market, where the market index ETF (BOVA11) performed similarly to fixed-income assets. This environment prompted a capital reallocation from equities to fixed income, a trend particularly noted in the Brazilian market (ANBIMA, 2024).

The effects of high and low volatility regimes on the average behavior of the assets' descriptive statistics can be observed in Figure 2.

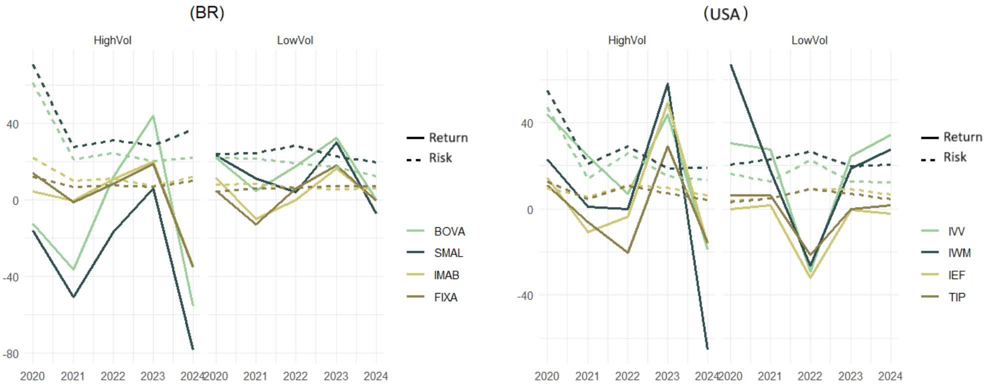


Figure 2: Mean and standard deviation of returns by year and volatility regime

Note: The annualized values for Return (mean) and Risk (standard deviation) are expressed in percentage terms. LowVol represents the low-volatility regime, and HighVol represents the high-volatility regime.

From the average returns and volatility for the two states classified into volatility regimes, a generally similar behavior is observed between the Brazilian and U.S.A. markets. In both markets, high-volatility years (HighVol) tend to present more variable and negative average annualized returns, while annual volatility tends to be higher. This behavior is expected, as periods of high market uncertainty often result in sharp declines in asset prices. On the other hand, in low-volatility years (LowVol), average annualized returns are more frequently positive, and annualized volatility is lower. Fixed-income-linked ETFs exhibit more defensive behavior, with lower volatility compared to equity-linked ETFs, for both regimes.

This dynamic reinforces the importance of regime-dependent allocation strategies, given that different assets react differently to changes in general market conditions. This approach has been widely discussed in the literature, as pointed out in recent studies (Campani, Garcia & Lewin, 2021; Li, Uysal & Mulvey, 2022; Nystrup et al., 2019; Oprisor & Kwon, 2021).

Table 5 presents the correlations between assets in high and low volatility regimes, as well as for the full period. The results confirm findings in the literature, as pointed out by Ang and Timmermann (2012), Davoodi, Fereydooni and Rastegar (2024), Jacobs and Karagozoglu (2014) and Peng, Kim and Mittnik (2022), indicating that the correlation between assets varies according to the regime. This change has direct implications for the dynamics of systemic risk and the effectiveness of portfolio diversification.

A key challenge in portfolio management is that asset correlations tend to increase during high-volatility environments, which, as pointed out by Kritzman, Page and Turkington (2012) and Jacobs and Karagozoglu (2014), undermines the effectiveness of traditional diversification. Our results confirm this, showing that the correlation between market-index and small-cap ETFs exceeded 90% during high-volatility periods in both the Brazilian and American markets. To counter this, regime-conditional diversification allows for dynamic adjustments that can reduce

Table 5: Asset correlation matrix by volatility regime and full period

Volatility Regime / Period			**Asset Pairs**			
BR	BOVA/SMAL	BOVA/IMAB	BOVA/FIXA	SMAL/IMAB	SMAL/FIXA	IMAB/FIXA
HighVol	0.91	0.56	0.57	0.57	0.57	0.66
LowVol	0.81	0.19	0.30	0.23	0.33	0.35
Full	0.87	0.44	0.43	0.45	0.45	0.51
USA	IVV/IWM	IVV/IEF	IVV/TIP	IWM/IEF	IWM/TIP	IEF/TIP
HighVol	0.92	-0.22	0.09	-0.16	0.16	0.67
LowVol	0.79	0.05	0.12	0.03	0.12	0.81
Full	0.87	-0.10	0.09	-0.07	0.13	0.74

Note: Full refers to 2020–2024. LowVol is low volatility; HighVol is high volatility.

losses during turbulent periods and optimize returns in market recoveries, thus generating gains over a full market cycle (Bae, Kim & Mulvey, 2014; Li, Uysal & Mulvey, 2022; Oprisor & Kwon, 2021).

Our rebalancing strategy is triggered only when the HMM forecasts a change in the market regime, an approach designed to avoid excessive transaction costs from frequent, unnecessary adjustments. Following a regime change, the asset weights are recalculated using the new estimated parameters to maximize the investor's utility by balancing expected return, risk, and transaction costs (Equation 3) within a long-only framework. The HMM results, by identifying high and low volatility regimes, are therefore used to determine the necessary weights between the assets.

Table 6: Average asset weights by volatility regime, risk aversion level, and transaction cost of 0.05%.

Market	λ	Regime	Assets				SumEquity	SumFixed
			BOVA	**SMAL**	**IMAB**	**FIXA**		
BR	5	HighVol	0.2680	0.2227	0.2631	0.2462	0.4907	0.5093
		LowVol	0.2721	0.1434	0.2705	0.3141	0.4155	0.5846
	10	HighVol	0.2517	0.1593	0.2753	0.3137	0.4110	0.5890
		LowVol	0.2387	0.1118	0.2822	0.3674	0.3505	0.6496
			IVV	**IWM**	**IEF**	**TIP**		
USA	5	HighVol	0.2829	0.3709	0.1764	0.1698	0.6538	0.3462
		LowVol	0.2765	0.2561	0.2172	0.2502	0.5326	0.4674
	10	HighVol	0.3655	0.2343	0.2150	0.1852	0.5998	0.4002
		LowVol	0.2559	0.2019	0.2514	0.2908	0.4578	0.5422

Note: Average weights represent mean asset allocation from 2020 to 2024, based on a Long Only strategy. Equity-linked ETFs are represented by Sum Equity, and fixed-income ETFs by Sum Fixed. The low-volatility regime is LowVol, and the high-volatility regime is HighVol. Investor risk aversion is represented by λ, and transaction costs are represented by γ.

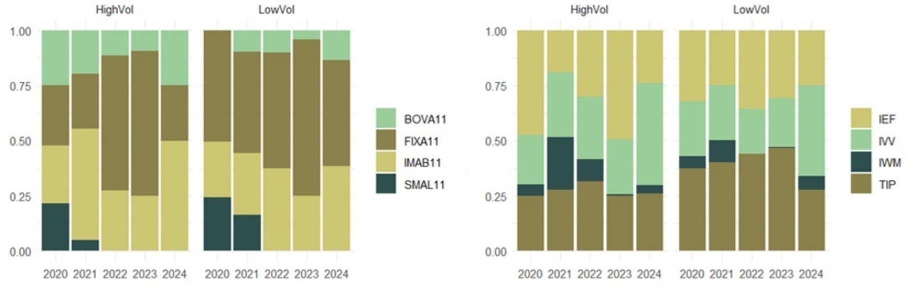
Although Table 6 presents the portfolio weights only for λ=5, λ=10, and ϕ=0.05%, as described in the methodological procedures, in this article λ also assumed the value 1, in order to evaluate the sensitivity of the portfolios to risk aversion. For the transaction cost parameter, values of 0.03% and 0.1% were also considered. The corresponding results were not included in Table Table 6 for reasons of space and organization. Furthermore, the results observed for these additional parameters were consistent with what was expected; that is, an increase in risk aversion tends to make the portfolio more conservative, and an increase in transaction costs inhibits variations in portfolio weights (DeMiguel, Garlappi & Uppal, 2009; Grant, Kwon & Satchell, 2024; Lewin & Campani, 2024; Li, Uysal & Mulvey, 2022).

The increase in λ resulted in a greater concentration of weights in fixed-income-linked ETFs, which is consistent with more conservative investor profiles. Although the behavior of the weights is as expected in relation to the assumed level of aversion, surprisingly, the average concentration in these assets was even higher during periods classified as low volatility, compared to high volatility periods.

For Brazil, the concentration between fixed and variable income was opposite to that observed for the United States. In Brazil, depending on the regime and the adopted risk aversion, the concentration in variable income did not exceed 50%, with allocation in fixed income reaching 65% in the LowVol regime and for more risk-averse agents. In the case of the United States, in general, a greater concentration in variable income was observed, not differing substantially from the 60/40 allocation indicated by Kandhari (2024) and Grant, Kwon and Satchell (2024), with the exception of the LowVol case that considers a more risk-averse investor. The average weights between ETFs, by regime and year, can be observed in Figure 3.

(a) BR – λ=5 e γ= 0.05%

(b) USA – λ=5 e γ= 0.05%



(c) BR – λ=10 e γ= 0.05%

(d) USA . – λ=10 e γ= 0.05%

Figure 3: Evolution of weights between regimes

However, this behavior can also be explained by the logic of the optimization model itself, which aims to maximize expected return and simultaneously minimize portfolio risk, while penalizing transaction costs (Equation 3). During periods of low volatility, even though the perceived risk in the stock market is lower, ETFs that replicate fixed-income securities may offer a more attractive risk-return ratio, particularly in specific economic contexts, such as those marked by high real interest rates.

During the period considered for asset allocation, 2020 to 2024, the benchmark interest rate in the Brazilian market increased by 10.25 percentage points, while in the U.S.A. market there was an increase of 4.5 percentage points. The upward movement of the benchmark interest rate exerts negative pressure on company valuations, as it raises the cost of capital and reduces the present value of future cash flows, resulting in a drop in stock prices and, consequently, a reduction in companies' market value (Kim, 2003). On the other hand, increases in the interest rate raise the profitability of fixed-income securities without necessarily altering the risk perception of these securities, making them relatively more attractive in terms of risk-return trade-off. The model, by capturing this favorable condition, naturally increases the allocation to these assets.

Conversely, during periods of high volatility, although a greater allocation in ETFs that replicate fixed-income indices might be expected, indicating a greater demand for assets with more predictability and security, the model may identify specific return opportunities in variable income assets, especially after significant price corrections, thus increasing allocation flexibility. Paradoxically, the portfolio may, on average, have a lower weight in fixed-income ETFs during these periods, suggesting that the model takes advantage of specific moments to capture short-term gains in variable income. Furthermore, at all times, in addition to weighing the return-risk trade-off, the model also considers the welfare loss from excessive turnover of portfolio positions. It should be noted that, during the analyzed period, the magnitude of the interest rate increase in the Brazilian market was greater compared to the American market, partly justifying the high average concentration in fixed-income-linked ETFs recommended by the model, regardless of the observed volatility regime. In the United States, however, the smaller magnitude of the interest rate cycle allowed for greater flexibility of the model in alternating between ETFs.

These results suggest, as in Chauvet, Senyuz and Yoldas (2015), a relationship between financial volatility and macroeconomic fluctuations, reinforcing the importance of an analysis of the macroeconomic and financial conditions associated with each regime. They also highlight the optimization model's ability to identify relevant nuances for dynamic asset allocation, surpassing traditional heuristic patterns, such as the 60/40 split frequently found in the literature.

Furthermore, as previously mentioned, the higher the transaction cost, the lower the model's predisposition to adjust portfolio weights. A cost of 0.1% proved to be excessive, resulting in almost constant allocations and severely restricting the flexibility of the regime-switching model. On the other hand, a cost of 0.03% allowed for more aggressive adjustments, which can generate high turnover and, consequently, accumulated costs over time. The intermediate cost of 0.05% provided an adequate balance, allowing for dynamic allocations without excessive changes, also indicating the importance of proper parameter calibration (Han, 2020).

Thus, the parameters that showed the most favorable balance between stability and flexibility were λ=5 or λ=10 and a transaction cost of 5 basis points (0.05%). Studies such as those by Boyd et al. (2017) and Li, Uysal and Mulvey (2022) point out that a risk aversion around 5 usually provides good portfolio stability.

The following portfolio allocation strategies were constructed using the HMM fitted with conditional volatility, which was identified as the most accurate proxy. Furthermore, the 252-day window was adopted for all subsequent analyses, as this timeframe was deemed the most appropriate for balancing the need for recent data with the robustness of the estimation, as justified in the previous section.

Table 7 presents a comparison of the performance of the allocation strategies tested for the Brazilian (BR) and North American (USA) markets. The evaluated metrics include Cumulative Return, Sharpe Ratio, Sortino Ratio, Maximum Drawdown, and Volatility, allowing for the evaluation of allocation conditional on regime switching and market uncertainty in the analyzed period (2020–2024).

In the Brazilian market, considering a risk aversion of 5 and a transaction cost of 0.05%, the Dynamic Regime strategy showed a cumulative return (-6.40%) superior to the Single Regime strategy but inferior to the Naive strategy (2%). This pattern repeats across the other indicators: the Dynamic Regime shows intermediate performance between the two strategies. With an increase in risk aversion to 10, the Dynamic Regime improves its return (1.06%) and begins to show the lowest volatility (15.37%). However, the Naive strategy continues to have a better return and lower drawdowns, remaining the most efficient in general terms. The Single Regime strategy, in turn, showed consistently inferior performance across all analyzed metrics.

In the North American market, the results favor the Dynamic Regime strategy. For λ=5, it shows

Table 7: Performance Comparison of Allocation Strategies

Market	λ	**Strategy**	**Cum. Return**	**Sharpe**	**Sortino**	**MaxDD**	**Volatility**
BR	5	Dynamic Regime	-6.40	-1.45	0.08	36.96	17.60
		Single Regime	-43.95	-5.42	-3.00	51.39	23.73
		EW	**2.00**	**-1.07**	**0.89**	**30.99**	**16.49**
	10	Dynamic Regime	1.06	**-1.03**	0.74	32.04	**15.37**
		Single Regime	-38.72	-4.95	-2.64	51.39	22.24
		EW	**2.00**	-1.07	**0.89**	**30.99**	16.49
USA	5	Dynamic Regime	**12.81**	-1.38	2.07	29.12	14.21
		Single Regime	-6.52	-5.27	0.15	41.30	18.12
		EW	12.52	**-0.85**	**2.15**	**23.88**	**12.46**
	10	Dynamic Regime	**16.02**	**-0.82**	**2.57**	24.30	12.67
		Single Regime	1.23	-4.72	0.78	35.08	15.73
		EW	12.52	-0.85	2.15	**23.88**	**12.46**

Note: Bold values indicate the best-performing indicators among the strategies for a given level of risk aversion and market. The analysis covers the period from January 2020 to December 2024. Cum. Return denotes cumulative return, MaxDD refers to maximum drawdown, and Volatility is annualized. All performance metrics are reported in percentage terms. The Sharpe ratio is the adjusted version proposed by Israelsen (2005).

a cumulative return of 12.81%, slightly higher than the Naive portfolio (12.52%). For λ=10, the Dynamic Regime stands out for its cumulative return (16.02%) and risk-adjusted return ratios (Sharpe and Sortino). Even so, the Naive portfolio continues to exhibit, slightly, lower volatility (12.46%) and a smaller drawdown (23.88%). The Single Regime strategy again shows the worst results, reinforcing its limitations in capturing market dynamics.

These results suggest that incorporating volatility regimes provides better risk control compared to the single-regime approach. For the analyzed period, the Dynamic Regime strategy showed superior performance in the North American market, proving competitive even against the Naive portfolio. In the Brazilian market, however, the strong performance of the equally weighted (1/n) strategy suggests that specific characteristics of this emerging market, such as high fundamental uncertainty, may limit the gains from more sophisticated models.

This finding aligns with literature arguing that while Mean-Variance models are suited for predictable risks, the simplicity of the 1/n portfolio makes it robust in highly uncertain markets where predictive power is limited and estimation errors are high (De Jong, 2018; DeMiguel, Garlappi & Uppal, 2009). According to this view, the effectiveness of naive strategies stems from their ability to handle inherent unpredictability without relying on explicit forecasts (Santiago & Leal, 2015). However, this simplicity entails an implicit cost, particularly a greater exposure to unmanaged tail risks and extreme events, aspects not fully captured by mean and variance (Hwang, Xu & In, 2018).

Crucially, our results reveal an important nuance that addresses this trade-off: the performance of the Dynamic Regime strategy improves as investor risk aversion increases, while the Naive portfolio's performance remains static. This demonstrates that even when not outperforming on raw returns, the adaptive approach adds significant value by allowing risk exposure to be aligned with the investor's profile. This corroborates the growing evidence that multi-regime models are particularly effective for dynamic asset allocation, enabling more precise adjustments to investment strategies compared to single-regime approaches (Fons et al., 2021; Li, Uysal & Mulvey, 2022).

5. Conclusion

This study addressed a critical gap in asset allocation by identifying the most effective volatility proxy for HMM-based regime detection. The main contribution is the empirical demonstration that conditional volatility, estimated by GARCH models, outperforms historical and implied proxies in both U.S.A. and Brazilian markets. This finding provides a clear methodological directive for constructing more accurate regime-switching models.

In asset allocation, the practical relevance of this finding was demonstrated through a dynamic strategy that showed superior risk control compared to the single-regime approach. It consistently outperformed other benchmarks in the U.S.A. market, especially for more conservative profiles. In the Brazilian market, however, a puzzle emerged: the naive $(1/n)$ strategy proved more resilient in terms of raw returns, challenging the superiority of the sophisticated model.

The resilience of the naive strategy in Brazil underscores the robustness of simple approaches in environments of high fundamental uncertainty, a known characteristic of some emerging markets. This highlights that while the correct proxy provides a better informational foundation, its final impact on performance also depends on specific market characteristics. Crucially, our results reveal the solution to this puzzle: the performance of the dynamic strategy improves as investor risk aversion increases, while the naive portfolio's performance remains static. This demonstrates that the adaptive model's primary value lies in its superior risk control. This is further evidenced by the portfolio's rational response to the macroeconomic context, a high concentration in fixed-income during a period of high real interest rates, an adaptation that traditional heuristics, like the 60/40 rule, do not capture.

Therefore, the main practical implication is that regime-based allocation is a valuable tool, particularly for risk-averse investors. The model's ability to offer more precise risk control and align the portfolio with an investor's utility profile provides a significant advantage that naive strategies cannot. Managers and investors can thus use conditional volatility as a robust input to build adaptive strategies designed to preserve capital and enhance risk-adjusted returns, even in challenging market environments.

Despite the robust results, this study has limitations that open new avenues for research. The methodological decision to use two regimes, while favoring interpretability, could be expanded to three or more states to capture more complex dynamics. It is also suggested to further explore the calibration of transaction cost and risk aversion parameters and to expand the analysis to other markets and asset classes, such as commodities and cryptocurrencies. Finally, a promising path would be to integrate the conditional volatility proxy validated here with a comparative analysis of different risk measures in the optimization function.

References

Aigner, A. A. (2022). A gaussian mixture hidden markov model for the vix. *Available at SSRN 4389763*.

ANBIMA. (2024). *Raio x do investidor brasileiro* [Acessado em março de 2025]. https://www.anbima.com.br/pt_br/especial/raio-x-do-investidor-brasileiro.htm

Ang, A., & Timmermann, A. (2012). Regime changes and financial markets. *Annu. Rev. Financ. Econ.*, *4*(1), 313–337.

Ardia, D., Bluteau, K., Boudt, K., & Catania, L. (2018). Forecasting risk with markov-switching garch models: A large-scale performance study. *International Journal of Forecasting*, *34*(4).

Ardia, D., Bluteau, K., & Rüede, M. (2019). Regime changes in bitcoin garch volatility dynamics. *Finance Research Letters*, *29*, 266–271.

Asgharian, H., Christiansen, C., & Hou, A. J. (2023). The effect of uncertainty on stock market volatility and correlation. *Journal of Banking & Finance*, *154*, 106929.

Bae, G. I., Kim, W. C., & Mulvey, J. M. (2014). Dynamic asset allocation for varied financial markets under regime switching framework. *European Journal of Operational Research*, 450–458.

Bai, Y., & Cai, C. X. (2024). Predicting vix with adaptive machine learning. *Quantitative Finance*.

Bekaert, G., & Hoerova, M. (2014). The vix, the variance premium and stock market volatility. *Journal of econometrics*, *183*(2), 181–192.

Boyd, S., Busseti, E., Diamond, S., Kahn, R. N., Koh, K., Nystrup, P., Speth, J., et al. (2017). Multi-period trading via convex optimization. *Foundations and Trends® in Optimization*, 1–76.

Campani, C. H., Garcia, R., & Lewin, M. (2021). Optimal portfolio strategies in the presence of regimes in asset returns. *Journal of Banking & Finance*, *123*, 106030.

Chauvet, M., Senyuz, Z., & Yoldas, E. (2015). What does financial volatility tell us about macroeconomic fluctuations? *Journal of Economic Dynamics and Control*, *52*, 340.

Davoodi, S., Fereydooni, A., & Rastegar, M. A. (2024). Can portfolio construction considering esg still gain high profits? *Research in International Business and Finance*, *67*, 102126.

De Jong, M. (2018). Portfolio optimisation in an uncertain world. *Journal of Asset Management*, *19*(4).

De Paula, L. F., Fritz, B., & Prates, D. (2024). The metamorphosis of external vulnerability from 'original sin'to 'original sin redux': Currency hierarchy and financial globalization in emerging economies. *Review of International Political Economy*, 1–28.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The review of Financial studies*, *22*(5), 1915.

Douglas, J. W., Conrad, J., & Frederick, J. A. (2025). Budgeting for the human-made disaster of war: Us aid to ukraine. *Public Money & Management*, *45*(3), 247–255.

Fereydooni, A., & Mahootchi, M. (2023). An algorithmic trading system based on a stacked generalization model and hidden markov model in the foreign exchange market. *Global Finance Journal*, *56*, 100825.

Fons, E., Dawson, P., Yau, J., Zeng, X.-j., & Keane, J. (2021). A novel dynamic asset allocation system using feature saliency hidden markov models for smart beta investing. *Expert Systems with Applications*, *163*, 113720.

Grant, A., Kwon, O. K., & Satchell, S. (2024). Properties of risk aversion estimated from portfolio weights. *Journal of Asset Management*, *25*(5), 427–444.

Haghani, V., & Dewey, R. (2016). A case study for using value and momentum at the asset class level. *The Journal of Portfolio Management*, *42*(3), 101–113.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the econometric society*, 357–384.

Han, C. (2020). How much should portfolios shrink? *Financial Management*, *49*(3), 707–740.

Hwang, I., Xu, S., & In, F. (2018). Naive versus optimal diversification: Tail risk and performance. *European Journal of Operational Research*, *265*(1), 372–388.

Israelsen, C. (2005). A refinement to the sharpe ratio and information ratio. *Journal of Asset Management*, *5*(6), 423–427.

Jacobs, M., & Karagozoglu, A. K. (2014). On the characteristics of dynamic correlations between asset pairs. *Research in International Business and Finance*, *32*, 60–82.

Kandhari, J. (2024, April). *Big picture: Return of the 60/40* (tech. rep.). Macro Insight/Morgan Stanley.

Kim, K.-h. (2003). Dollar exchange rate and stock price: Evidence from multivariate cointegration and error correction model. *Review of Financial economics*, *12*(3), 301–313.

Kolm, P. N., Tütüncü, R., & Fabozzi, F. J. (2014). 60 years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research*, *234*(2), 356.

Kritzman, M., Page, S., & Turkington, D. (2012). Regime shifts: Implications for dynamic strategies (corrected). *Financial analysts journal*, *68*(3), 22–39.

Lewin, M., & Campani, C. H. (2024). Estratégias restritas ótimas para investimentos baseados em fatores no mercado de ações brasileiro. *Revista Contabilidade & Finanças*, *35*, e2051.

Li, X., Uysal, A. S., & Mulvey, J. M. (2022). Multi-period portfolio optimization using model predictive control with mean-variance and risk parity frameworks. *European Journal of Operational Research*, *299*(3), 1158–1176.

Ma, F., Lu, X., Yang, K., & Zhang, Y. (2019). Volatility forecasting: Long memory, regime switching and heteroscedasticity. *Applied Economics*, *51*(38), 4151–4163.

MacLean, L., Yu, L., & Zhao, Y. (2022). A generalized entropy approach to portfolio selection under a hidden markov model. *Journal of Risk and Financial Management*, *15*(8), 337.

MacLean, L., Zhao, Y., & Zhang, O. (2025). Mean-variance optimization with inferred regimes. *Annals of Operations Research*, 341–368.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*, 77–91.

Matos, G. A. d. S., Iquiapaza, R. A., & Ferreira, B. P. (2014). Análise da exposição a perdas dos etfs brasileiros conforme as técnicas de avaliação de risco de mercado value at risk (var) e expected shortfall (es). *BBR-Brazilian Business Review*, *11*(4), 91–115.

Megaritis, A., Vlastakis, N., & Triantafyllou, A. (2021). Stock market volatility and jumps in times of uncertainty. *Journal of International Money and Finance*, *113*, 102355.

Michaud, R. O. (1989). The markowitz optimization enigma: Is 'optimized'optimal? *Financial analysts journal*, *45*(1), 31–42.

NEFIN. (2017). *Metodologia nefin – núcleo de estudos em finanças* (tech. rep.) (Acessado em janeiro de 2025). FEA-USP. São Paulo, Brasil.

Nguyen, N., & Nguyen, D. (2015). Hidden markov model for stock selection. *Risks*, *3*(4), 455.

Nystrup, P., Boyd, S., Lindström, E., & Madsen, H. (2019). Multi-period portfolio selection with drawdown control. *Annals of Operations Research*, *282*(1), 245–271.

Nystrup, P., Hansen, B. W., Madsen, H., & Lindström, E. (2015). Regime-based versus static asset allocation: Letting the data speak. *The Journal of Portfolio Management*, *42*(1), 103–109.

Nystrup, P., Madsen, H., & Lindström, E. (2018). Dynamic portfolio optimization across hidden market regimes. *Quantitative Finance*, *18*(1), 83–95.

Oprisor, R., & Kwon, R. (2021). Multi-period portfolio optimization with investor views under regime switching. *Journal of Risk and Financial Management*, *14*(1), 3.

Peng, C., Kim, Y. S., & Mittnik, S. (2022). Portfolio optimization on multivariate regime-switching garch model with normal tempered stable innovation. *Journal of Risk and Financial Management*, *15*(5), 230.

Santiago, D. C., & Leal, R. P. C. (2015). Carteiras igualmente ponderadas com poucas ações e o pequeno investidor. *Revista de Administração Contemporânea*, *19*, 544–564.

S&P Dow Jones Indices. (2024, July). *Metodologia do índice s&p/b3 ibovespa vix* (tech. rep.) (Acessado em janeiro de 2025). S&P Global. Nova York, EUA.

Tu, J. (2010). Is regime switching in stock returns important in portfolio decisions? *Management Science*, *56*(7), 1198–1215.

Uysal, A. S., & Mulvey, J. M. (2021). A machine learning approach in regime-switching risk parity portfolios. *The Journal of Financial Data Science*, *3*(2), 87–108.

Zucchini, W., & MacDonald, I. L. (2009). *Hidden markov models for time series: An introduction using r*. Chapman; Hall/CRC.