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EFFICIENCY AND FRACTALITY IN THE BRAZILIAN STOCK MARKET: MEASURING THE IMPACTS OF COVID-19 ON A SECTOR-LEVEL ANALYSIS

LEANDRO DOS SANTOS MACIEL
FACULDADE DE ECONOMIA, ADMINISTRAÇÃO E CONTABILIDADE DA UNIVERSIDADE DE SÃO PAULO - FEA

EDUARDO NORIO ADACHI
FACULDADE DE ECONOMIA, ADMINISTRAÇÃO E CONTABILIDADE DA UNIVERSIDADE DE SÃO PAULO - FEA

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1. Introduction

Proposed by Fama (1965), the Efficient Market Hypothesis (EMH) is one of the most important theoretical frameworks in finance, used as the basis for the development of several pricing models and decision-supporting systems. The EMH essentially assumes that asset prices reflect all relevant information, which are available to all market participants. Many studies in the empirical finance literature have investigated the validity of the EMH, especially its weak-form, which states that prices changes follow a random walk dynamics, thus implying the unpredictability of security returns based on historical data (Markiel & Fama, 1970; Titan, 2015).

Generally, asset returns do not follow the weak-form of the EMH, as the random walk dynamic is considered very restrictive. In the literature, multifractal detrended fluctuation analysis (MF-DFA) provides a new tool for the analysis of asset returns efficiency. This method is commonly employed by econophysics for examining financial time series stylized facts such as multifractality, long-memory dependences, asymmetry, persistence, etc, features that can be related to market inefficiency (Al-Yahyaee, Mensi, Ko, Yoon, & Kang, 2020). Differently from the traditional rescaled range analysis (R/S), which measures time series variability, MF-DFA does not suffer from spurious results due to time series non-stationary and short-term dependencies, being a more robust technique to describe time series long-range autocorrelations Ali, Shahzad, Raza, and Al-Yahyaee (2018). Instead of answering whether or not a series follows a random walk process (adherence of the weak-form of the EMH), MF-DFA has an additional advantage by providing the ranking of the assets (time series) in terms of their in(efficiency) degree through the spectrum of the generalized Hurst exponents.

In this context, this paper aims to evaluate the adherence to the weak-form of market efficiency in the Brazilian stock market using MF-DFA to measure equity shares level of fractality. Empirical analysis comprises all stocks negotiated at the Brazilian stock exchange, B3. Efficiency is measured for all the stocks and analyzed for each industry sector. In addition, it is also an objective the evaluation of the recent Corona Virus Disease (COVID-19) pandemic on the B3 degree of efficiency. The stock returns of different industries may exhibit heterogeneous responses to the COVID-19 pandemic depending on the type of business. Hence, measuring the degree of prices (in)efficiency in a period of such a black swan event, especially at an industry-level, is of great relevance for market participants and policy makers. Finally, the relation between efficiency and industry sectors provides empirical findings regarding how efficiency differs according to the business in Brazil.

The main contributions are described in the following. First, this work consists in an extensive empirical experiment focusing on all stocks traded at the Brazilian stock market instead of using a single series. MF-DFA literature generally used stock indices to evaluate a particular market. However, it is a biased selection as the shares composing these indices are associated with a higher liquidity. As liquidity is a key factor on price efficiency, this selection may provide biased results with a sample concerning equities of common price efficiency properties. Second, the paper addresses the discussion concerning the level of efficiency at an industry-level in Brazil, which is new to the best of our knowledge. Third, this paper is a first look on the impacts of COVID-19 pandemic on the efficiency of the Brazilian stock market, which is of particular relevance for policymakers, market operators, and individual investors. Many authors have indicated the negative impacts on financial markets efficiency due to COVID-19 (Naeem, Farid, Ferrer, & Shahzad, 2021; Ozkan, 2021; Mnif, Jarboui, & Mouakhar, 2020), hence the research addresses this discussion concerning the Brazilian stock market. Additionally, literature is still lacking on studies using MF-DFA for the Brazilian markets in general.

After this introduction, this paper proceeds as follows. Section 2 provides a literature review on the evaluation of the weak-form of market efficiency, especially using MF-DFA analysis and concerning the impacts of the COVID-19 pandemic. The methodology is detailed in Section 3. Results are provided in Section 4, followed by the conclusion in Section 5.

2. Literature review

The empirical finance literature provides a plethora of studies investigating the Efficient Market Hypothesis (Fama, 1965, 1970), especially its weak-form, stating that securities' price returns follow a random walk dynamic and are not influenced by past events. Hence, future trends can not be anticipated using historical data (Fama, 1970). Notice that a market is not necessarily informational inefficient under the rejection of the random walk hypothesis, as the markets are also influenced by agents expectations and microstructure noises.

A consensus in the literature concerning the empirical validity of the EMH is still not verified. Results are influenced by the methodological approaches, the markets evaluated, data frequency, and the period used for evaluation. In many cases, the heterogeneity of inferences is observed, as different methods may diverge on the confirmation of the EMH for the same data. Examples of empirical validation on the adherence of the weak-form of market efficiency are found for the following economies: United States (Pesaran & Timmermann, 1995); Indonesia, Malaysia, Singapore and South Korea (Rizvi & Arshad, 2014); Canada (Alexeev & Tapon, 2011); United Arab Emirates (Marashdeh & Shrestha, 2008); Iran (Oskooe, Li, & Shamsavari, 2010); India, Pakistan, Bangladesh, and Siri Lanka (Shahzad, Zakaria, Ali, & Raza, 2018); Bulgaria, Czech Republic, Slovakia and Hungary (Hasanov & Omay, 2007).

These works generally used statistical tests to evaluate the weak-form of market efficient. An alternative approach is the use of fractal analysis, particularly the multifractal detrendend fluctuation analysis. Cajueiro and Tabak (2004, 2005) and Matte, Aste, and Dacorogna (2005) are examples that used monofractal methods to rank and compare efficiency of different markets. However, monofractal techniques are not appropriate as they may result in spurious inferences (Kwapien, Oswie, & Drozdz, 2005; Pasquini & Serva, 1999). Hence, the MF-DFA, proposed by Kantelhardt et al. (2002), provides a flexible and efficient approach to test multifractal properties (long-memory) in nonlinear time series (Mensi, Tiwari, & Yoon, 2017).

Traditional nonlinear variance ratio tests or autocorrelation functions are not able to identify multifractal structures. Fractal properties are associate to time series that present heavy tails and long-memory. As these features are commonly observed in financial asset price returns (stylized facts), the use of MF-DFA appears as a suitable technique to evaluate random walk properties in such series, as stated by the econophysics literature (Arshad, Rizvi, & Ghani, 2016; Ali et al., 2018; Tiwari, Aye, & Gupta, 2019).

Several studies evaluated the weak-form of market efficient using MF-DFA. For instance, Arshad et al. (2016) tested the random walk dynamics for the stock markets of the Organization for Islamic Cooperation. Authors stated that information efficiency increases over time. Using MF-DFA and Multifractal Detrended Cross-Correlation Analysis (MF-DXA), Mensi, Hamdi, Shahzad, Shafiullah, and Al-Yahyaee (2018) provided evidence on the change in the inefficiency levels for Saudi Arabian banks. Similar findings were also observed by Sukpitak and Hengpunya (2016) for Thailand in the period from 1975 to 2015. Similar examples are the works of Dewandaru, Masih, Bacha, and Masih (2015) for the Islamic equities traded in the U.S. Dow Jones Islamic market, Tiwari, Albulescu, and Yoon (2019) for US exchange traded funds (ETF) indices, Shahzad, Nor, Mensi, and Kumar (2017) for the US credit and stock markets at the industry level, Ali et al. (2018) for the equity markets of Jordan, Malaysia, Pakistan and Turkey, and Al-Yahyaee et al. (2020) considering cryptocurrencies.

Recent studies also evaluated the impacts of COVID-19 on financial markets efficiency. Ozkan (2021) investigated the stock markets from the United Kingdom (UK), United States (US), Spain, Italy, Germany, and France. Using wild bootstrap variance ratio test on daily data, results indicated that equities returns are not adherent to the weak-form of market efficiency. Deviations from market efficiency are seen more in the stock markets of the US and UK during the COVID-19 outbreak than in other stock markets. Additionally, the findings indicated the presence of abnormal returns during the COVID-19 pandemic.

Diniz-Maganini, Diniz, and Rasheed (2021) examined the price efficiency and net cross-correlations among Bitcoin, gold, a US dollar index, and the Morgan Stanley Capital International World Index (MSCI World) after the pandemic outbreak. Using intraday data and detrended partial cross-correlation analysis, the authors found that Bitcoin prices were more efficient than the US dollar and MSCI World indices. On the other hand, based on multifractal analysis, Mnif et al. (2020) provided empirical results proving that COVID-19 has a positive impact on the cryptocurrency market efficiency.

Still considering cryptocurrencies, Naeem, Bouri, Peng, Shahzad, and Vo (2021) examined the asymmetric efficiency of 1-hour data of Bitcoin, Ethereum, Litecoin, and Ripple. Based on asymmetric MF-DFA, a significant asymmetric multifractality in the price of cryptocurrencies was found and that upward trends exhibit stronger multifractality than downward trends. Results also indicated that the COVID-19 outbreak negatively affected the efficiency of the associated digital coins, as a substantial increase in the levels of inefficiency was observed. Also using asymmetric MF-DFA, Naeem, Farid, et al. (2021) confirmed the presence of asymmetric multifractality in the green and traditional bond markets, and verified that inefficiency in both bond markets significantly escalated during the COVID-19 outbreak.

Impacts of COVID-19 pandemic on market efficiency were also evaluated for foreign exchange rates (Aslam, Aziz, Nguyen, Mughal, & Khan, 2020), for gold and oil prices (Mensi, Sensoy, Vo, & Kang, 2020), for a number of sectors in the US stock market (Choi, 2021), and for markets that are top oil producers and oil consumers (Mensi, Lee, Vinh Vo, & Yoon, 2021). Hence, this paper addresses this discussion concerning the Brazilian stock market, including a sector-level analysis.

3. Multifractal detrended fluctuation analysis

Multifractal detrendend fluctuation analysis, proposed by Kantelhardt et al. (2002), uses generalized Hurst exponents and is a powerful tool for detecting multifractality in a time series. Properties like persistence, anti-persistence and random walk behavior can be measured through MF-DFA. Particularly for asset returns series, according to the values of the q-th order Hurst exponents, the adherence of the weak-form of market efficiency can be evaluate, as well as the measuring of the corresponding level of (in)efficiency.

Let $\{r(t)\}$, for $t = 1,2,\ldots,T$, be a non-stationary time series of length T. In this work, $r(t) = \ln[P(t)] - \ln[P(t-1)]$ are the log-returns, and P_t stands for the equity price at t. To apply the MF-DFA technique, a new sequence, denoted by $y(t)$, called the profile function, is constructed as:

$$y(t) = \sum_{k=1}^{i} [r(k) - \bar{r}], \;\; i = 1,2,\ldots,T, \tag{1}$$

where $\bar{r} = (1/T)\sum_{t=1}^{T} r_t$.

The time series $y(t)$ is divided into $T_s \equiv \mathrm{int}(T/s)$ windows of equal length s, where s is the scale parameter. These segments must be non-overlapping. As the length T may not be a multiple of the scale parameter s, the constructed intervals may disregard a short part of the profile function. Thus, the subdivision is performed from the opposite end and a total $2T_s$ sub-intervals is constructed (Tiwari, Aye, & Gupta, 2019; Bai & Zhu, 2010). This mechanism avoids any information lost.

For each window, $\{\nu = 1,\ldots,2T_s\}$, the next step comprises the fitting of a polynomial of order m (usually, $m = 1$) using least squares to compute the local tendency. Then the variance is calculated for $\nu = 1,\ldots,T_s$ and $\nu = T_s+1,\ldots,2T_s$, respectively:

$$F^2(s,\nu) = \frac{1}{s}\sum_{t=1}^{s}\{y[(\nu-1)s+t]-y_\nu(t)\}^2, \tag{2}$$

$$F^2(s,\nu) = \frac{1}{s}\sum_{t=1}^{s}\{y[T-(\nu-T_s)s+t]-y_\nu(t)\}^2, \tag{3}$$

where y_ν corresponds to the fitting polynomial in the ν-th segment.

The q-th order fluctuation function, $F_q(s)$, is obtained by averaging over all segments:

$$F_q(s) = \left\{\frac{1}{2T_s}\sum_{\nu=1}^{2T_s}[F^2(s,\nu)]^{q/2}\right\}^{1/q}. \tag{4}$$

The q-order Hurst exponent is defined as the slopes $h(q)$ of regression lines for each q-order root mean square $F_q(s)$. The order q, $q \in \Re, q \neq 0$, encompasses the effect of varying degrees of fluctuation on $F_q(s)$, thus it is related to small (larger) fluctuations when $q < 0$ ($q > 0$). Notice that the standard DFA is obtained when $q = 2$ (Tiwari, Aye, & Gupta, 2019).

From the fluctuation function, the final step of MF-DFA comprises the computing of the scale index, bases on the log-log plots of $F_q(s)$ against s for each value of q. A linear pattern in the log-log scale is obtained if $F_q(s)$ is in accordance to the called power-law:

$$F_q(s) \sim s^{h(q)}. \tag{5}$$

A fluctuation function value $F_q(s)$ is computed for each segment s. The q-order generalized Hurst index, $h(q)$, corresponds to the slope of $ln\left(F_q(s)\right) \sim ln(s)$. The dependence of $h(q)$ on q provides relevant informations concerning the patters of a time series (Ali et al., 2018). A time series is said monofractal if $h(q)$ does not depend on q. Otherwise, a series is multifractal when $h(q)$ depends on q and monotonically decreases as q increases.

If the Hurst exponent is equal to 0.5, $h(q) = 0.5$, the time series is a random walk independent process. In such a case, the stochastic process is adherent to the weak-form of market efficiency. Fractal market hypothesis also called the corresponding return process when $h(q) = 0.5$ as informationally effective (Münnix et al., 2012). Finally, when $0 < h(q) < 0.5$ ($0.5 < h(q) < 1$) the time series correlation are anti-persistence (persistence), indicating the rejection of a random walk dynamics.

MF-DFA also provides a mechanism to rank the series according to their levels of multifractality. A market level of (in)efficiency is measured by a market deficiency measure (MDM), computed as (Tiwari, Aye, & Gupta, 2019):

$$\text{MDM} = \frac{1}{2}\left(|h(q_{min})-0.5|+|h(q_{max})-0.5|\right), \tag{6}$$

where q_{min} and q_{max} are, respectively, the minimum and maximum orders of the fluctuation function (Tiwari, Aye, & Gupta, 2019; Ali et al., 2018; Mensi et al., 2017).

If a series follows the random walk hypothesis (weak-form of market efficiency), $h(q) = 0.5$ for distinct values of q. Hence, a market is said weak-form efficient when MDM is zero. Otherwise, the market is inefficient and the higher the MDM value the higher its inefficiency. This property provides a mechanism to rank different markets in terms of (in)efficiency, and also allows the measure of efficiency over time. Thus, based on the MDM values, the level of efficiency can be evaluated for the periods before and after the COVID-19 pandemic and across industries (at a sector-level), which are the main objectives of this work.

4. Empirical analysis

To evaluate the adherence of the Brazilian stock market to the weak-form of the efficiency market hypothesis, including a sector-level analysis, it is measured how much the dynamic of the asset returns distances from a random walk process. For each stock, the level of efficiency is calculated using MF-DFA, which allows the ranking of the equities in terms of efficiency using a market deficiency measure - MDM in Eq. (6). The whole sample is divided into two sub-samples: considering the periods before and after the COVID-19 pandemic. In addition, the level of adherence to the weak-form of efficiency is related to the corresponding shares industry sectors. All the analyses in this paper were implemented using R software.

4.1 Data

The data is a collection of daily closing prices of all traded stocks at the B3, the Brazilian stock exchange, considering the period from January 2, 2015 to May 31, 2022, with a total of 1,833 observations (data were collected at Economatica). It was considered only the equities associated with a positive trading volume during the entire sample. Common and preferred shares of a sample company were also included. The final sample comprised 135 stocks. Due to space limitations the list of stock tickers are available under request.

The analyses are based on two sub-samples: before and after COVID-19 pandemic. This division is made to provide the evaluation of the potential impacts of the pandemic outbreak on the B3 stocks efficiency, in general, and concerning the industry-level consequences. Before COVID-19 sub-sample concerns data from January 2, 2015 to December 30, 2019, with a total of 1,234 observations. After COVID-19 sample includes data from June 1, 2017 to May 31, 2022, with a number of 1,236 daily closing prices. The initial data point was selected to keep in the samples five years of daily prices, which is generally used by the MF-DFA literature as historical data number of observations. Finally, the ending point considers the most recent data when the analyses were performed. Both sub-samples are relatively equal in terms of the number of observations to avoid the effect of the sample size when analyzing the COVID-19 systemic crisis impacts using MF-DFA.

4.2 Fractality of B3 stock returns

To evaluate the fractality of the stocks traded at the B3, MF-DFA was applied to all the 135 assets considered in this work using data from the two sub-samples: before and after the COVID-19 outbreak. Table 1 shows the main statistics of the generalized Hurst exponents, $H(q)$, for different values of the fluctuation order, q. As suggested by Rizvi and Arshad (2014), the scaling range assumed the values of $s_{min} = 10$ and $s_{max} = (T/4)$, where T is the series' number of observations.

Tab. 1: Main statistics of the generalized Hurst exponents ($H(q)$) for different values of the fluctuation function order (q) computed for the returns of the stocks traded at B3. Data samples comprise the periods before and after the COVID-19 outbreak.

q	Average	Std. Dev.	Maximum	Minimum	1st Quartile	2nd Quartile	3rd Quartile
Panel A: before COVID-19 pandemic.							
-4	0.6531	0.1507	1.4656	0.4156	0.5490	0.6061	0.7410
-3	0.6312	0.1451	1.3900	0.3987	0.5332	0.5880	0.7191
-2	0.6060	0.1367	1.2596	0.3803	0.5121	0.5717	0.6861
-1	0.5774	0.1250	1.0255	0.3610	0.4870	0.5513	0.6574
0	0.5459	0.1152	0.9037	0.3327	0.4644	0.5268	0.6229
1	0.5154	0.1107	0.8683	0.2744	0.4388	0.5026	0.5865
2	0.4858	0.1085	0.8254	0.2127	0.4156	0.4770	0.5430
3	0.4584	0.1077	0.7808	0.1556	0.3873	0.4572	0.5158
4	0.4342	0.1077	0.7405	0.1092	0.3636	0.4342	0.4937
Panel B: after COVID-19 pandemic.							
-4	0.6653	0.1131	1.1923	0.4372	0.5961	0.6486	0.7216
-3	0.6425	0.1084	1.1210	0.4218	0.5714	0.6253	0.6976
-2	0.6154	0.1015	0.9942	0.3896	0.5480	0.5989	0.6668
-1	0.5832	0.0933	0.8553	0.3387	0.5221	0.5694	0.6384
0	0.5459	0.0887	0.8121	0.2791	0.4883	0.5390	0.6004
1	0.5034	0.0876	0.7757	0.2168	0.4471	0.4958	0.5641
2	0.4559	0.0899	0.7417	0.1581	0.4065	0.4434	0.5090
3	0.4086	0.0947	0.7151	0.1073	0.3496	0.3981	0.4702
4	0.3673	0.0996	0.6920	0.0660	0.3048	0.3602	0.4304

A series is monofractal if $H(q)$ is constant for all q. On the other hand, series is multifractal, fractal structure varies according to the measurement conditions. The negative and positive q values are used to explain the effects of the small and large variations, respectively. Based on the results from Table 1, for both sub-samples, all statistics of the generalized Hurst exponents ($H(q)$) change in accordance to the values of q. This provides empirical evidence on the mutifractality of the B3 asset returns for both sub-samples considered: before and after the pandemic outbreak. Hence, price returns from the equities traded at the B3 do present multifractality behavior, concerning both the sample periods analyzed.

The evidence of multifractality is fundamental for investors to identify specific price patterns and thus predict the future movements, as this behavior contradicts the weak-form of the efficiency market hypothesis (Wang, Liu, Gu, Cao, & Wang, 2010; Cajueiro & Tabak, 2004). Multifractal dynamics is due to different long-range correlations of the small and large fluctuations in the time series (Kantelhardt et al., 2002). The presence of a correlation structure in the returns series distinguishes themselves from a random walk, assumed by the weak-form of efficiency, a hypothesis that is rejected for the Brazilian equity market.

From Table 1, it is possible to notice that the level of multifractality is different for the periods before and after COVID-19 pandemic. Most of the generalized Hurst exponents statistics changed considerably between the two sub-samples. For example, the average values of the Hurst exponent for $q=-4$ and $q=4$ are 0.65 and 0.43, respectively, during the period before the COVID-19. Concerning the period after the pandemic, these values are 0.66 and 0.36 (see Table 1), respectively. Hence, the variation of the generalized Hurst exponent ($\Delta H = |H(-4) - H(4)|$) is higher for the period after the sanitary crisis, indicating that the pandemic accentuated the multifractal behavior of the Brazilian stock market.

According to MD-DFA method, if $H(2) > 0.5$ a series is persistent or has long-memory. In this case, larger (smaller) fluctuations are likely to be followed by larger (smaller) variations. Persistence increases as the greater the value of $H(2)$. Otherwise, series is called anti-persistent if $H(2) < 0.5$, where large (small) values are more likely to be followed by small (large) values. If $H(2) = 0.5$, the series follows a random walk process and is entirely stochastic in nature.

Table 1 shows that the average and the median (2nd quartile) of $H(2)$ indicate that most of the stocks are anti-persistent or negative correlated, as $H(2) < 0.5$, for both periods (before and after the pandemic). It means that large (small) returns are more likely to be followed by small (large) returns. Hence, a tendency to return to a long-term mean is verified for the considered stock returns. On the other hand, some Brazilian equities are persistent or have long-memory, as the values of $H(2)$ for the 3rd quartile are higher than 0.5 for both sub-samples considered (see Table 1).

Finally, results from Table 1 indicated that the average and the median values of the Hurst exponents when $q = 2$ are different from 0.5 ($H(2) \neq 0.5$), implying that most of the stocks traded at B3 do not follow a random walk process in nature, for both periods, before and after the COVID-19 outbreak. It confirms that the Brazilian stock market is not efficient in terms of the weak-form of efficiency.

To further illustrate the fractal behavior of the B3 asset returns, for each sample (before and after COVID-19), it is selected the equities associated with the lowest and the highest variation in the generalized Hurst exponent according to the extreme order average values, i.e., $\Delta H = |H(-4) - H(4)|$. Lower (higher) variations of the Hurst exponent are associated with a lower (higher) level of fractality for the asset returns or, in other words, with a higher (lower) efficiency in terms of the weak-form of market efficiency.

Concerning the data before the COVID-19 outbreak, the stocks related with the lower and higher Hurst exponent variation, ΔH, are, respectively, WEGE3 and CPFE3. WEGE3 corresponds to the ticker of the ordinary shares of WEG, which is a company operating worldwide in the electric engineering, power and automation technology areas. WEG produces electric motors, generators, transformers, drives and coatings. CPFE3 is the ticker of ordinary shares of CPFL Energia, group of electric energy generation and distribution in Brazil. Figure 1 shows the MF-DFA findings for WEGE3 stock returns for the period before the COVID-19 outbreak.

Figure 1-a illustrates the fluctuation function $\log_2(F_q(s))$ versus $\log_2(s)$ plot - $q = -4$ (black), $q = 0$ (red), and $q = 4$ (green). It can be observed that the local slope of the plots changes with crossover time scales, which is an evidence of mutifractality, even for WEGE3, the stock associated with the lower variation of the Hurst exponent - with a behavior closer to a monofractal structure. Multifractality is also confirmed by the dependence of the Hurst exponent on the values of q, as verified in Figure 1-b: as q increases, $H(q)$ shows a downward trend. When a time series exhibit monofractality, the generalized Hurst exponent should not vary with q.

Figure 1-c provides the Renyi exponent $\tau(q)$ over different values of q. The monofractal and white-noise time series has a mass exponent $\tau(q)$ with a linear q-dependency. The linear q-dependency of $\tau(q)$ leads to a constant h_q of these time series because h_q is the tangent slope of $\tau(q)$. In contrast, the multifractal time series has mass exponents $\tau(q)$ with a curved q-dependency, which is the case of WEGE3 returns and, consequently, a decreasing singularity exponent h_q (see Figure 1-a). Finally, Figure 1-d shows the multifractal of the Hölder spectrum f_α versus the Hölder exponent, α. If a time series is monofractal, then f_α would reduce to the Hurst exponent, such that $\alpha = H$, and $f_\alpha = 1$. Concerning WEGE3 returns, the multifractal spectrum shows an inverted parabola shape, which validates our previous results of multifractality (see Figure 1-d).

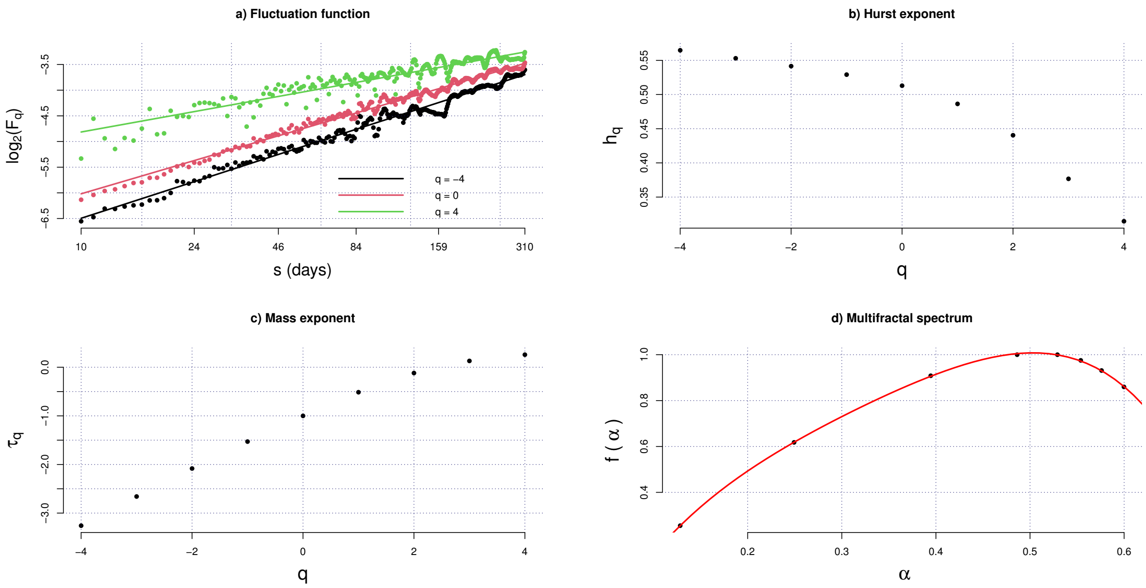

Fig. 1: The MF-DFA results of WEGE3 price returns using data for the period before the COVID-19 pandemic. a) Fluctuation functions for $q = -4$, $q = 0$, $q = 4$. b) Generalized Hurst exponent for each q. c) Mass exponent, $\tau(q)$. d) Multifractal spectrum.

Figure 1-c provides the Renyi exponent $\tau(q)$ over different values of q. The monofractal and white-noise time series has a mass exponent $\tau(q)$ with a linear q-dependency. The linear q-dependency of $\tau(q)$ leads to a constant h_q of these time series because h_q is the tangent slope of $\tau(q)$. In contrast, the multifractal time series has mass exponents $\tau(q)$ with a curved q-dependency, which is the case of WEGE3 returns and, consequently, a decreasing singularity exponent h_q (see Figure 1-a). Finally, Figure 1-d shows the multifractal of the Hölder spectrum f_α versus the Hölder exponent, α. If a time series is monofractal, then f_α would reduce to the Hurst exponent, such that $\alpha = H$, and $f_\alpha = 1$. Concerning WEGE3 returns, the multifractal spectrum shows an inverted parabola shape, which validates our previous results of multifractality (see Figure 1-d).

Figure 2 shows the MF-DFA results for CPFE3 stock returns for the period before COVID-19, which is the equity related with the higher variation of the Hurst exponent (ΔH_q) in this period, i.e. the asset with a higher fractal behavior, less adherent to the weak-form of market efficiency.

In contrast to the results from WEGE3 (Figure 1), a higher multifractality pattern of CPFE3 is verified by: the larger slopes of the fluctuation function (Figure 2-a); the larger sensitivity of the Hurst exponent to the change in the values of q (Figure 2-b); a more curved q-dependency of the Renyi exponent $\tau(q)$ over different values of q (Figure 2-d); and the most well defined U-shaped form of the multifractal spectrum function (Figure 2-d). The comparison between Figures 1 and 2 reveals that both stocks, WEGE3 and CPFE3, do present multifractal behavior, which means that they are not weak-form efficient stocks (do not follow a random walk dynamics). However, multifractality is more relevant for CPFE3, and this characteristic does provide a mechanism to rank different time series in terms of weak-form (in)efficiency.

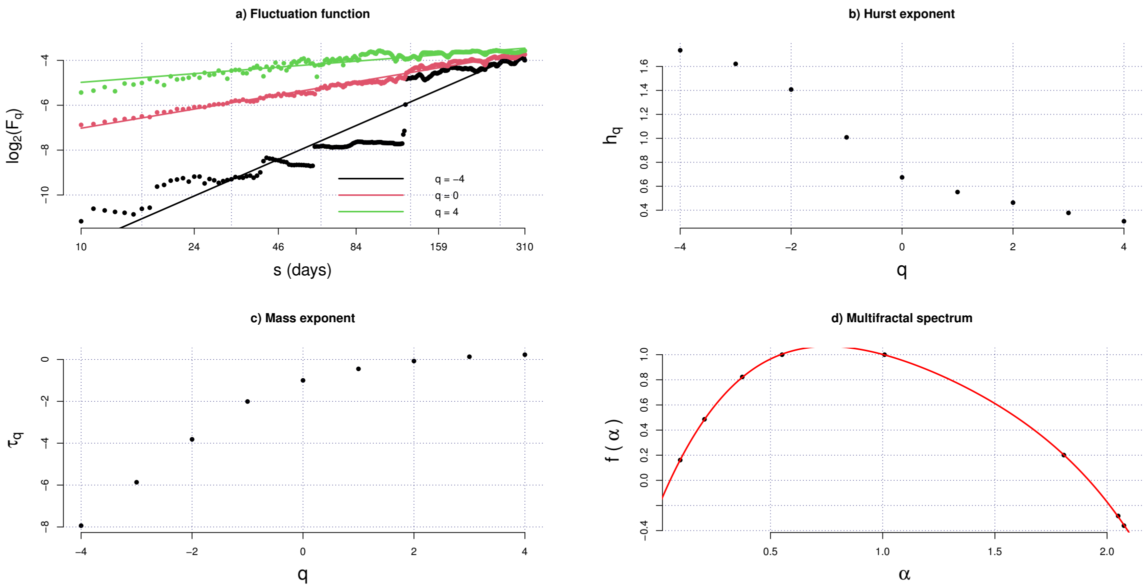

Fig. 2: The MF-DFA results of CPFE3 price returns using data for the period before the COVID-19 pandemic. a) Fluctuation functions for $q=-4$, $q=0$, $q=4$. b) Generalized Hurst exponent for each q. c) Mass exponent, $\tau(q)$. d) Multifractal spectrum.

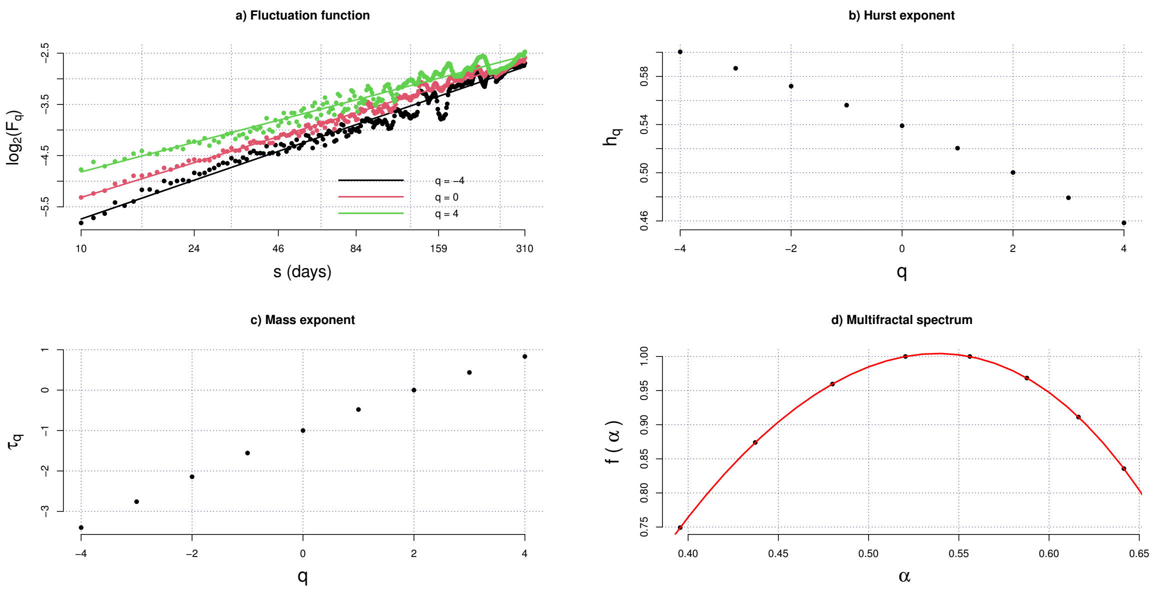

Fig. 3: The MF-DFA results of AMER3 price returns using data for the period after the COVID-19 pandemic. a) Fluctuation functions for $q=-4$, $q=0$, $q=4$. b) Generalized Hurst exponent for each q. c) Mass exponent, $\tau(q)$. d) Multifractal spectrum.

Similarly, Figures 3 and 4 show, respectively, the MF-DFA findings for AMER3 and KEPL3 stock returns for the period after COVID-19 crisis. These stocks are the ones associated with the lower and higher variations of the Hurst exponent (ΔH_q), respectively, i.e. the most and the less efficient equities for the period covering the COVID-19 outbreak. AMER3 and KEPL3 are the tickers of ordinary shares of Lojas Americanas and Kepler Weber, respectively. Lojas Americanas is a retail chain founded in 1929 and is the sixth largest retailer in Brazil. Kepler Weber is a agricultural company that provides storage and post-harvest solutions in South

America. Again, Figures 3 and 4 indicate that both stocks returns of AMER3 and KEPL3 are multifractal, and that this behavior is more evident for KEPL3.

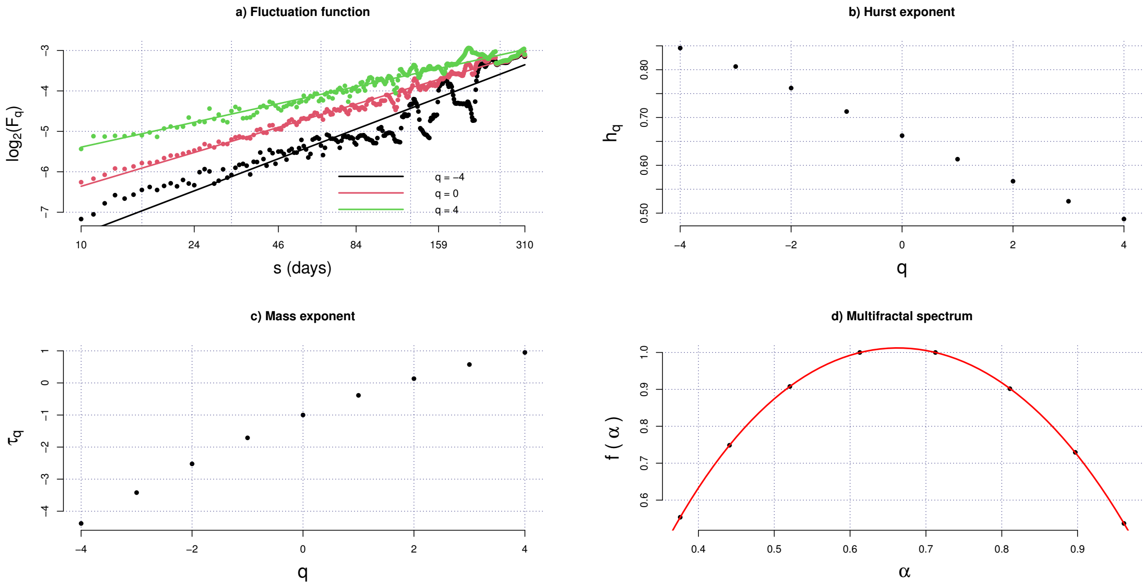

Fig. 4: The MF-DFA results of KEPL3 price returns using data for the period after the COVID-19 pandemic. a) Fluctuation functions for $q = -4$, $q = 0$, $q = 4$. b) Generalized Hurst exponent for each q. c) Mass exponent, $\tau(q)$. d) Multifractal spectrum.

4.3 Efficiency and COVID-19 pandemic

The richer the multifractality, the higher is the degree of market inefficiency because of long-range autocorrelation and fat-tail properties. Due to the evidence of multifractality in the Brazilian equity market, to measure and rank the market efficiency of the considered stocks, the MDM measure was calculated - as in Eq. (6). A MDM value closer to zero indicates a weak-form efficient market. The larger the MDM value the more inefficient is a market (multifractality beahvior is higher).

Table 2 provides the descriptive statistics of MDM values calculated for the 135 equities concerning both sub-samples: before and after the COVID-19 outbreak. For both samples, it is possible to verify that all stocks are not efficient in terms of the weak-form of market efficiency: MDM values differ from zero (returns do not follow a random walk dynamics). Even the minimum value of MDM, which is related to the least inefficient stock, is higher than zero. Concerning the minimum, maximum and standard deviation values, it is worth to notice that the stocks traded at B3 do present different levels of (in)efficiency (see Table 2). Further, regarding the effects of the COVID-19 pandemic, the average and all quartile statistics of MDM values for the period after the pandemic are higher than the corresponding ones when the sample comprises data before the sanitary crisis (Table 2). This result means that the Brazilian equity market became more inefficient after the crisis, i.e. the multifractal behavior of asset returns is more prominent as a consequence of the pandemic.

Figure 5 presents the histogram of the MDM values calculated for the 135 stocks negotiated at B3 using data before and after the COVID-19 outbreak. First of all, for both sub-samples, equities traded at the B3 do have different levels of (in)efficiency, and the right tail side of both the distributions reveal the significant presence of more inefficient stocks.

Tab. 2: Market Deficiency Measure (MDM) statistics calculated for asset price returns of 135 stocks traded at B3 during the periods before and after the COVID-19 pandemic.

Sample	Average	Std. Dev.	Maximum	Minimum	1st Quartile	2nd Quartile	3rd Quartile
Before COVID-19	0.6274	0.0759	1.0272	0.5549	0.5806	0.6002	0.6494
After COVID-19	0.6540	0.0512	0.9501	0.5650	0.6221	0.6485	0.6741

Concerning the pre-COVID period, the histogram is clearly right-skewed, revealing that most of the equities had similar MDM values (are equally inefficient), but some of the assets tend to be more inefficient, i.e. are associated with higher MDM values (fat right tail) - see Figure 5 top panel. On the other hand, the bottom histogram of Figure 5 clearly reveals the impact of COVID-19 pandemic on the efficiency of Brazilian stock market. The distribution became less asymmetric, still right-skewed, but the distribution is clearly dislocated to the right, when compared to the top histogram. Hence, the COVID-19 pandemic had a significant impact on the level of efficiency of the Brazilian stock market or, in other words, the Brazilian stock market observed a decline in the efficiency degree after crisis.

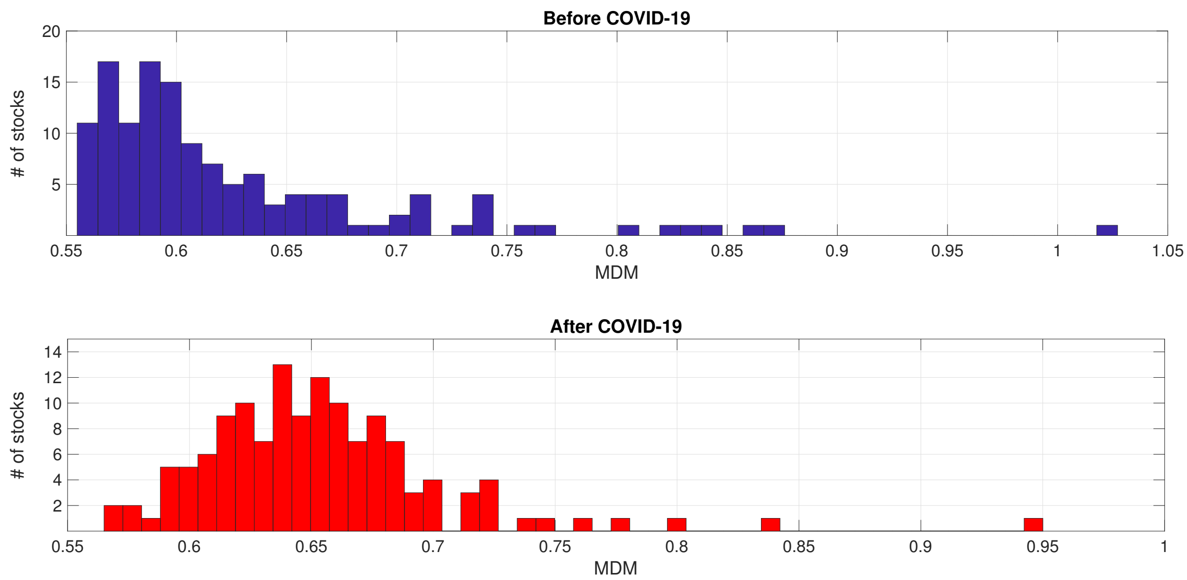

Fig. 5: MDM histograms for the stocks traded at B3. Data samples comprise the periods before and after the COVID-19 outbreak.

The analyses of MDM indicate that Brazilian stock returns do not follow a random walk dynamics (are multifractal), rejecting the weak-form of market efficiency. If the random walk hypothesis is rejected, it means that returns are not random and can be predicted. As a consequence, asset prices do have different dynamic properties, including fat-tail and long-term correlations, being predictable.

4.4 Efficiency and industry sectors

This study also provides the evaluation of the potential impacts of the pandemic outbreak on the B3 stocks efficiency for six industry sectors, which are the most relevant (or have more than 10 stocks) from the sample considered in this paper, namely: cyclical consumption (30), non-cyclical consumption (11), financial (25), industrial goods (17), basic materials (15), and public utilities (19). Values in parenthesis correspond to the number of stocks from the whole sample in each sector. Cyclical consumption is related to companies that usually respond to variations in the economy status, such as clothes industry, civil constructions, etc. Non-cyclical consumption is the companies that are not directly related to variations in the economy. This is because many of their products are considered essential (food, personal hygiene, health, etc). Financial market is mostly constituted by financial institutions and banks. Industrial goods is associated with industrial production of machinery. Basic materials comprises the companies that work with the extraction, production and commercialization of raw materials. Finally, public utility is related to companies operating in the generation and distribution of energy and in the basic sanitation.

Figure 6 and 7 show the histograms of the MDM values of each sector considering the periods before and after COVID-19 outbreak, respectively. The red line indicates the corresponding sample average. Most of industry sector histograms are clearly right-skewed in the pre-COVID period. On the contrary, Figure 7 illustrates the impact of the COVID-19 pandemic on the efficiency of the Brazilian industry sectors. Most of the distributions are right-skewed, became less asymmetric. Further, distributions in Figure 7 are clearly dislocated to the right, when compared to the pre-COVID histograms (Figure 6). Cyclical consumption is the industrial sector that had the biggest impact on distribution due to the COVID-19 outbreak: clear change from a right-skewed distribution to a more symmetric one with a lower variability. Basic materials is the only industry sector that the average did not increase in the after COVID-19 period, although the distribution showed a lower variability.

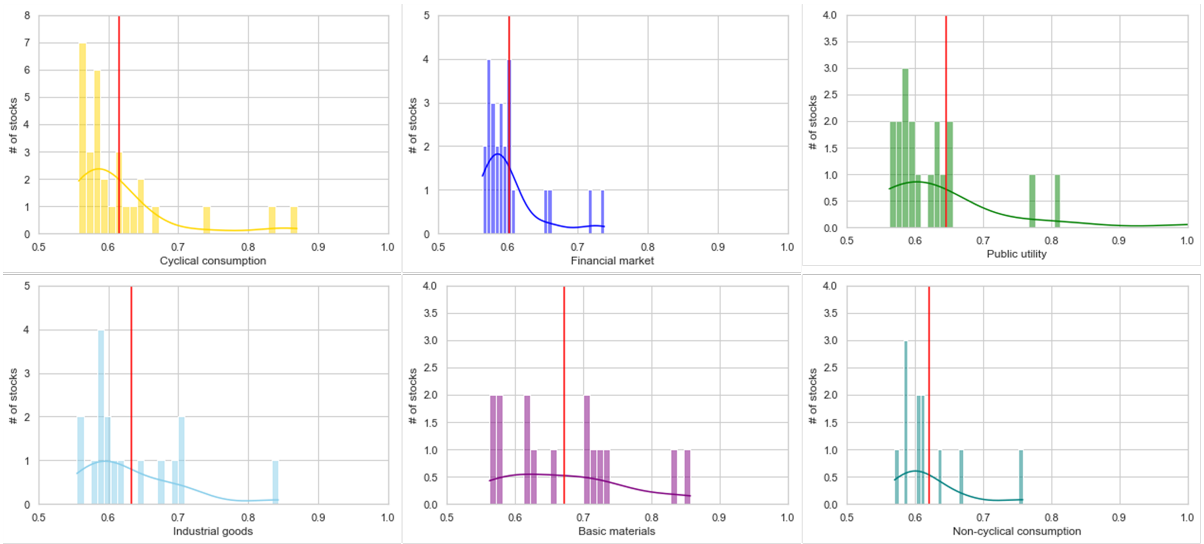

Fig. 6: MDM histograms for the stocks traded at B3 grouped by industry sectors for the period before the COVID-19 outbreak.

Table 3 shows the main statistics of the MDM values segmented by the industry sectors. $\Delta = (MDM_a/MDM_b) - 1$ stands for the associate variation, MDM_b and MDM_a are the MDM values for the periods before and after the pandemic, respectively. For both periods, it is possible to attest that all industry sectors are not efficient in terms of the weak-form of market efficiency: MDM values are greater than zero. The average of Cyclical consumption, Financial market, Public utility, Industrial goods, and Non-cyclical consumption are higher than the corresponding ones when the sample comprises data before the sanitary crisis. Basic materials is the only industrial sector that the average MDM is lower after COVID-19. Moreover, the majority of standard deviation and quartile statistics of MDM values for the period after the pandemic are higher than the corresponding ones concerning the data before the pandemic. These results implies that each industrial sector became more (in)efficient in different degrees after the crisis.

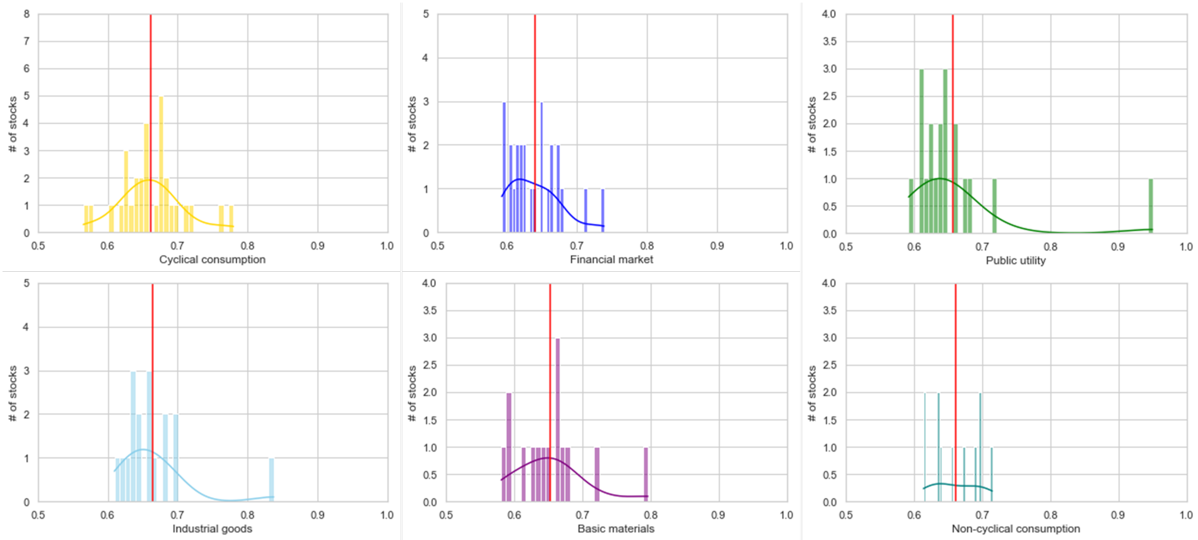

Fig. 7: MDM histograms for the stocks traded at B3 grouped by industry sectors for the period after the COVID-19 outbreak.

Tab. 3: Market Deficiency Measure (MDM) statistics calculated for asset price returns of 135 stocks traded at B3 during the periods before and after the COVID-19 pandemic grouped by industry sectors.

Statistics	Cyclical consumption			Financial market			Public utility		
	Before	After	Δ	Before	After	Δ	Before	After	Δ
Average	0,6153	0,6614	7,49%	0,6024	0,6391	6,10%	0,6454	0,6575	1,86%
Std. Dev.	0,0754	0,0463	-38,68%	0,0440	0,0371	-15,65%	0,1130	0,0770	-31,84%
1st Quartile	0,5689	0,6343	11,49%	0,5763	0,6118	6,16%	0,5860	0,6197	5,75%
2nd Quartile	0,5863	0,6555	11,81%	0,5897	0,6357	7,81%	0,6002	0,6420	6,96%
3rd Quartile	0,6218	0,6821	9,70%	0,6016	0,6611	9,90%	0,6435	0,6602	2,60%
Statistics	Industrial goods			Basic materials			Non-cyclical consumption		
	Before	After	Δ	Before	After	Δ	Before	After	Δ
Average	0.6330	0.6647	5.02%	0.6719	0.6533	-2.78%	0.6209	0.6607	6.41%
Std. Dev.	0.0734	0.0518	-29.42%	0.0919	0.0545	-40.70%	0.0531	0.0351	-33.82%
1st Quartile	0.5879	0.6364	8.25%	0.5994	0.6205	3.52%	0.5879	0.6366	8.28%
2nd Quartile	0.6026	0.6554	8.75%	0.6578	0.6507	-1.09%	0.6069	0.6568	8.23%
3rd Quartile	0.6793	0.6837	0.64%	0.7196	0.6702	-6.88%	0.6252	0.6926	10.78%

Table 4 provides the ranking of industrial sectors in terms of their inefficiency degree through MDM values before and after the pandemic. For example, Basic materials were the most inefficient industrial sector pre-COVID. Although, Industrial goods became the most inefficient industrial sector after COVID-19. These results demonstrates that each industrial sector behave different in relation of efficiency in a period of a black swan event, like COVID-19, and market participants and policy makers may have to consider this differences in their strategies.

Tab. 4: Industry sector inefficiency ranking before and after the COVID-19 pandemic.

Industrial sectors	MDM		Inefficiency ranking	
	Before	After	Before	After
Basic materials	0,6719	0,6533	1	5
Public utility	0,6454	0,6575	2	4
Industrial goods	0,6330	0,6647	3	1
Non-cyclical consumption	0,6209	0,6607	4	3
Cyclical consumption	0,6153	0,6614	5	2
Financial market	0,6024	0,6391	6	6

5. Conclusion

This study empirically investigated the adherence of the Brazilian stock market to the weak-form of market efficiency, including the analysis concerning the industry sectors individually. It is also a first look at the impact of the COVID-19 outbreak on the Brazilian stocks efficiency concerning the entire market and the corresponding sectors, as different industries may exhibit heterogeneous responses to the COVID-19. Analyses were conducted using multifractal detrended fluctuation analysis (MF-DFA) on prices daily data of 135 stocks traded at the B3, the Brazilian stock exchange, during the period from January 2015 to May 2022.

The results demonstrated that the Brazilian stock market is multifractal, deviating from the weak-form of market efficiency, before and after the COVID-19 pandemic. After the COVID-19 outbreak, B3 efficiency have changed. Precisely, the stocks traded at B3 became generally more inefficient, indicating a negative impact of the sanitary crisis on the market efficiency for the evaluated Brazilian equity shares. Moreover, industry sectors provide different degrees of market efficiency, before and after the COVID-19 pandemic. However, with exception of the basic materials, the pandemic decreases the industries efficiency in general.

The findings of this study provide important implications. First, Brazilian stock prices display inefficient behavior during the evaluated period, which brings out the possibility to forecast future pricing movements based on historical information. Thus, financial decisions that assumes the random walk hypothesis must be thoughtfully revised, especially when important theoretical financial models are based on this assumption. This is quite relevant during the COVID-19 phase, where the stocks became less efficient. Regulation in these markets appears more important and necessary during the pandemic in order to reduce the negative aspects of the crisis. For instance, results may help policymakers shape a comprehensive response to improve B3 market efficiency.

Second, it was found that the level of efficiency in the Brazilian stock market is associated with the industry sector. This association could help policymakers, market operators, and individual investors to adapt theirs strategies accordingly the degree of efficiency of each industry. Future works include the analysis of the time varying dynamics of efficiency, and the consideration of asymmetric MF-DFA to capture the impacts on efficiency due to up and down market trends.

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