05, 06, 07 e 08 de novembro de 2024
ISSN 2177-3866

Applying Time Series Models to Risk Management in the Cryptocurrency Market

DANIEL PEREIRA ALVES DE ABREU
UNIVERSIDADE FEDERAL DE MINAS GERAIS (UFMG)

OCTÁVIO VALENTE CAMPOS
UNIVERSIDADE FEDERAL DE MINAS GERAIS (UFMG)

Agradecimento à orgão de fomento:
FAPEMIG CAPES CNPQ

Aplicação de Modelos de Séries Temporais para a Gestão de Risco do Mercado de Criptomoedas

Applying Time Series Models to Risk Management in the Cryptocurrency Market

RESUMO

Este estudo investigou o uso de modelos ARMA-GARCH para gerenciamento de risco em criptomoedas, focando nas 10 principais moedas com mais de 5 anos de histórico, excluindo stablecoins. Foram realizadas extensivas análises de variações de modelagens ARMA-GRACH para identificar modelos que minimizassem erros médios absolutos, obtendo MAPEs inferiores a 1%. Os resultados indicaram taxas de sucesso superiores a 95% na estimativa de Value at Risk (VaR) e Expected Shortfall (ES), sendo o último o mais preciso em certos cenários. A inclusão de variáveis externas como Fear and Greed Index (FGI) e Market Value to Realized Value (MVRV) melhorou a performance do VaR para várias criptomoedas, embora o ES externo tenha sido mais eficaz apenas para o Bitcoin. A pesquisa também destacou a necessidade de adaptação dos modelos às características individuais de cada criptomoeda, dada a complexidade na seleção de parâmetros GARCH e distribuições de erros. Esses achados sublinham a importância de abordagens personalizadas para mitigação de riscos em um ambiente de mercado altamente volátil e diversificado como é do caso dos criptoativos.

Palavras-Chave: Criptoativos; Gestão de Risco; Value-at-Risk; Expected Shortfall

ABSTRACT

This study investigated the use of ARMA-GARCH models for risk management in cryptocurrencies, focusing on the top 10 currencies with more than 5 years of history, excluding stablecoins. Extensive variance analyses of ARMA-GRACH modelling were carried out to identify models that minimised mean absolute errors, obtaining MAPEs of less than 1%. The results indicated success rates of over 95 per cent in estimating Value at Risk (VaR) and Expected Shortfall (ES), the latter being the most accurate in certain scenarios. The inclusion of external variables such as Fear and Greed Index (FGI) and Market Value to Realised Value (MVRV) improved VaR performance for several cryptocurrencies, although external ES was more effective only for Bitcoin. The research also highlighted the need to adapt the models to the individual characteristics of each cryptocurrency, given the complexity in selecting GARCH parameters and error distributions. These findings underline the importance of personalised approaches to risk mitigation in a highly volatile and diversified market environment such as cryptoassets.

Keywords: Cryptocurrencies; Risk Management; Value-at-Risk; Expected Shortfall

1 INTRODUCTION

The current market scenario is marked by technological innovations, increased speed of information dissemination and the emergence of new products and services. The financial market, in particular, has undergone significant transformations driven by the internet, which has facilitated interactions and the sharing of information at reduced costs (Suryono et al., 2020).

One of the most notable innovations in the last decade has been the creation of cryptocurrencies, a new decentralised financial product that can be identified as a hybrid between a currency, commodities and a stock (Charfeddine et al., 2020). Bitcoin, developed by Nakamoto (2008) and launched in 2009, was responsible for inaugurating this new branch of the financial market and, due to its gain in popularity, several other cryptocurrency systems began to be developed and commercialised, known as altcoins.

The rapid growth and global dissemination of cryptocurrencies has aroused academic interest, seeking to explain and understand the evolution of this new financial segment as well as finding ways to calibrate the addition of these new assets to investments.

The aim of this study is to measure the risk of these assets by means of the Value-at-Risk (VaR) and Expected Shortfall (ES) of the 10 largest cryptocurrencies. To do this, volatility forecasting techniques using GARCH models were used.

Three points stand out as differentiating features of this study: firstly, the use of 63 variations of models from the GARCH family to produce VaR and ES, allowing the identification of the most suitable models for forecasts based on historical data; secondly - instead of using only the univariate data of the series - two external variables were also included, the Fear and Greed Index (FGI) and the Market Value to Realized Value (MVRV), with the aim of verifying whether adding these variables to the GARCH models enables better modelling of the volatility of the time series; finally, the number of cryptocurrencies analysed in the study was increased to 10, whereas empirical studies usually focus on 3-5 currencies.

2 THEORETICAL FRAMEWORK

2.1 Cryptocurrencies

According to Lánský (2017), since the creation of Bitcoin by Nakamoto (2008), several investors have become interested in this new type of asset, mainly due to its unique characteristics such as:

1. Being decentralised, i.e. not regulated by governments, banks or companies;

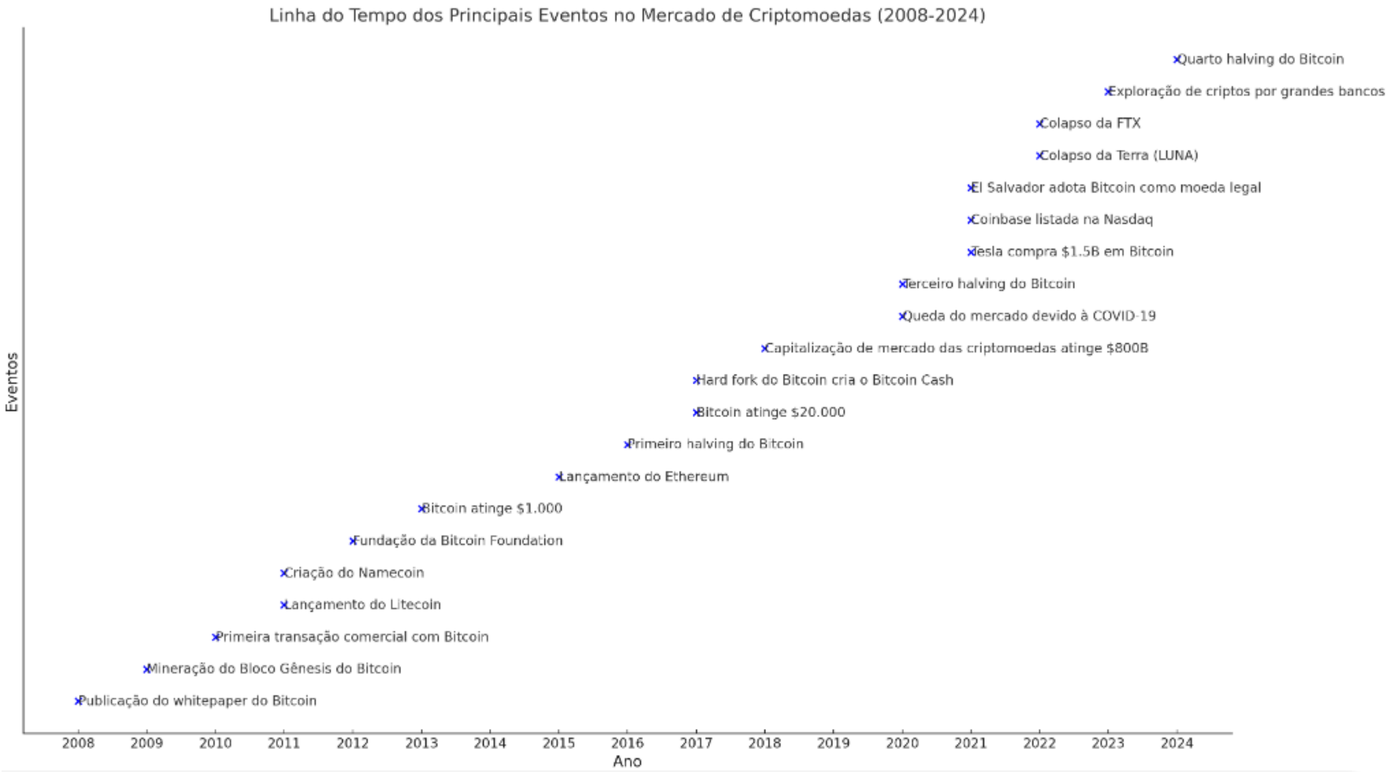
2. The realisation of peer-to-peer transactions, eliminating the need for intermediaries and consequently part of the transaction costs;
3. Use cryptographic techniques to secure transactions and protect the integrity of the network, as well as making transactions virtually anonymous between the parties;
4. Blockchain system, which records all transactions made in a public ledger in order to avoid possible fraudulent transactions.

Due to Bitcoin's rise in popularity and the spread of the blockchain system, new cryptocurrencies have been created and spread around the world, known as altcoins. These are generally designed to change some aspect of the system offered by Bitcoin. The main changes they propose concern the speed of transactions, changes to the mining system and the volume of availability (Gandal & Halaburda, 2014).

However, as Wang et al. (2020) point out, the high degree of volatility in these assets ended up creating a barrier to investors making stable gains. With this in mind, the so-called stablecoins were created, cryptocurrencies that aim to peg their prices to currencies (Theter, BitUSD and Nubits, for example) or even commodities (HelloGold, DigixDAO and Xaurum, for example).

Although this market is relatively new, it is still booming. In this sense, several new cryptocurrencies are created all the time, and in May 2024 there were a total of 10,043 cryptocurrencies, totalling a capitalisation of around USD 2.5 trillion. Of this amount, around 54.28 per cent corresponds to Bitcoins, 15.44 per cent to Etherium, 4.57 per cent to Tether, 3.53 per cent to Binance and 3.19 per cent to Solana. Finally, Figure 1 shows the timeline of the main events related to this market.

Figure 1 - Main Events in the Cryptocurrency Market



Source: The autors

In general, one of the most important events for this market is Bitcoin's halving periods. This process occurs every time 210,000 blocks are added to the blockchain and affects the amount paid for the mining process, thus limiting the creation of more Bitcoins. Initially, the mining process was paid 50 BTC and with each halving this amount is divided by two. A total of four such processes have already taken place, the first in 2012, one in 2016 and another in 2020, with the first half of 2024 being the last recorded halving.

Other important recent events for these markets include: the ban on cryptocurrency mining and transactions in China, causing a significant drop in prices; the collapse of LUNA, UST, crypto hedge fund Three Arrows Capital and FTX, which affected the degree of confidence in the cryptocurrency market; and the creation of CBDCs (Central Bank Digital Currencies), digital currencies created by central banks, which in principle goes against the decentralisation aspect of cryptocurrencies.

2.2 Risk Measures

Risk is essentially the possibility that the actual return on an investment will differ from the expected return. Markowitz (1952), a pioneer in modern portfolio theory, defines risk in terms of the volatility of returns, measured by metrics such as standard deviation and variance, which quantify the dispersion of results around the expected average. In this sense, risk management becomes an essential tool for making investment decisions, aimed at helping investors maximise their returns adjusted to a certain degree of risk that is compatible with their preferences.

According to Acereda et al. (2020), Value-at-Risk (VaR) is one of the traditional measures most commonly used to measure the risk of an asset. It can be calculated using Equation 1, in which μ and σ are the mean and standard deviation of the asset in question and Z_α is the critical value of the normal distribution corresponding to the confidence level α .

$$VaR_\alpha = \mu + \sigma * Z_\alpha \quad (1)$$

In this way, this metric helps to identify the maximum loss value of an asset given an error tolerance. It is worth noting that VaR can also be calculated based on historical data or via simulation models.

However, as criticised by Acerbi and Tasche (2002), this metric may not be the most suitable for analysing risk as it is limited to identifying the maximum potential loss. The authors therefore present Expected Shortfall (ES) as a more assertive alternative for risk management.

Also known as Conditional Value-at-Risk (CVaR), it differs from traditional VaR in that ES estimates the average loss in worst-case scenarios in addition to VaR. In other words, ES calculates the average loss that exceeds VaR, providing a more complete view of extreme risk (Acerbi & Tasche, 2002; Artzner et al., 1999). In general terms, Equation 2 exemplifies the ES calculation method.

$$ES_\alpha = \mathbb{E}[X|X > VaR_\alpha] = \frac{1}{1-\alpha}\int_0^{1-\alpha} VaR_\alpha \quad (2)$$

It is worth noting that these methodologies originally defined asset returns as belonging to a normal distribution. However, empirical studies such as those by Malek et al.(2023) , Fung et al. (2022) Acereda et al. (2020) and Conlon and McGee (2020), identifying leptokurtic properties and asymmetries incompatible with Gaussian models, opted to use alternative distributions to describe cryptoasset returns, such as the stable alpha distribution, Student's t, Azzalini-Skew-T and inverse normal. In addition, these studies identified clusters of return volatility, so ARMA-GARCH models were used to model the heteroscedasticity of cryptocurrency returns.

2.3 Previous studies

In general, studies on the cryptoasset market can be divided into three groups:

1. Analysing the characteristics of this market, comparing cryptocurrencies with financial assets such as commodities, shares and gold (Baur et al., 2018; Cai et al., 2023; Charfeddine et al., 2020);
2. Analysing the efficiency of this market (Abreu et al., 2022; Kristoufek & Vosvrda, 2019; Urquhart, 2016);
3. Analysing the volatility of these assets and their hedging capacity (Baur & Dimpfl, 2018; Hasan et al., 2024; Trimborn & Härdle, 2018).

Of these three typologies, this study falls into the third group. In order to study the hedging properties of stablecoins, Wang et al. (2020) analysed the relationship between two groups of these cryptocurrencies, one linked to the dollar and the other to gold, and Bitcoin, Litecoin and Ripple using a GARCH model with dynamic conditional correlations (dccGARCH) by Engle (2002).

As a result, the authors identified that stablecoins pegged to the dollar had a greater hedging capacity than the three traditional cryptocurrencies. Subsequently, in order to empirically verify this fact, portfolios were generated combining Bitcoin, Litecoin or Ripple with one of the stablecoins. Finally, the VaRs and ES of the portfolios were analysed, conforming to the hypothesis discussed above.

In order to identify more efficient alternatives for modelling the volatility of Bitcoin, Litecoin, Ripple and Etherium, Acereda et al. (2020) estimated the ES using traditional GARCH modelling and three of its variations: component GARCH (csGARCH), non-linear GARCH (nGARCH) and threshold GARCH (tGARCH).

The conclusions identified a better fit for the models when considering that cryptocurrency returns would follow an Azzalini-Skew-T distribution (AST), which is an extension of the t-distribution that incorporates an additional parameter to control asymmetry. Furthermore, it was concluded that csGARCH and nGARCH models provided more assertive predictions for cryptocurrencies, especially Bitcoin.

The study by Fung et al. (2022) draws an interesting conclusion about forecasting and risk analysis models. Analysing 8 different models from the GARCH family, the authors found that of the 254 cryptocurrencies analysed, around 1/3 of them had their returns best explained by a tGARCH model. However, when analysing the VaR of the returns, the authors report that the choice of an appropriate distribution for modelling the errors is more important for the accuracy of the VaR than the GARCH model itself. This reinforces similar conclusions, such as those of

Troster et al. (2019) and Ngunyi et al. (2019) on the importance of choosing distribution assumptions for modelling cryptocurrencies.

Using data from Bitcoin, Etherium, Ripple, Litecoin and Bitcoin Cash, Malek et al. (2023) proposed analysing changes in the VaR and ES of these cryptocurrencies before, during and after the COVID-19 pandemic. Their results signalled that the stable alpha distribution seems to be a viable alternative for estimating VaR and ES as well as the increase in downside risk of these assets after the start of the pandemic. This second conclusion corroborates the similar study carried out previously by Conlon and McGee (2020).

Finally, Huang et al. (2024) analysed the volatilities of Bitcoin, Etherium and Binance returns using GARCH modelling and concluded that in only 6, 3 and 2 days, respectively for each asset, the VaR identified were underestimated. Thus, the authors conclude the validity of applying GARCH modelling to cryptocurrency volatility analysis, but at the same time recognise that the use of VaR may not be sufficient to capture extreme volatility. This conclusion is in line with the idea of using ES in conjunction with VaR to analyse these types of assets.

3 METHODOLOGY

3.1 Database

For this study, data was collected on the dollar quotations of 10 cryptocurrencies from 05/14/2019 to 05/14/2023, thus totalling 5 years for the analyses. The assets chosen for the sample were based on the market capitalisation of the cryptocurrencies, excluding stablecoins and cryptocurrencies that did not exist throughout the 5-year period.

The following cryptocurrencies were therefore selected for this study: Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Dogcoin (DOGO), Binance (BNB), Cardano (ADA), Tronix (TRX), Bitcoin Cash (BCH), Chainlink (LINK) and Litecoin (LTC). The price data for these assets was collected via Yahoo Finance and the logarithmic returns for each series were calculated from this data.

In addition, data was collected on two series of external variables, the Fear and Greed Index (FGI) and the Market Value to Realised Value (MVRV), both obtained via BGeometrics. The FGI is calculated using a combination of indicators such as volatility, market volume, social media analysis, surveys, Bitcoin dominance and Google search trends. MVRV is the ratio between Bitcoin's current market value and realised value, which takes into account the price of each Bitcoin at the time it was last moved.

3.2 Time Series Models

In order to calculate VaR and ES, it was decided to use GARCH models, which not only make it possible to calculate forecasts of asset returns, but can also be used to make forecasts of the variance σ_t^2 of these assets. The traditional ARMA(p,q)-GARCH(r,m) model is calculated from Equation 3, in which the first part corresponds to the autoregressive and moving average effects of the returns and the second to the conditional heteroscedasticity effects of variance.

$$R_t = \phi_0 + \sum_{i=1}^{p} \phi_i R_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

$$Var(\varepsilon_t) = \sigma_t^2 = \alpha_0 + \sum_{k=1}^{r} \alpha_k \varepsilon_{t-k}^2 + \epsilon_t + \sum_{l=1}^{m} \beta_l \sigma_{t-l}^2 \qquad (3)$$

As studies of the limitations of the GARCH model have progressed, alternatives have been developed, aimed above all at increasing the robustness of the forecasts. To illustrate the modelling used in this study, Table 1 summarises the modifications made to each of the 6 variations selected.

Table 1 - GARCH Modelling Variations Used in the Study

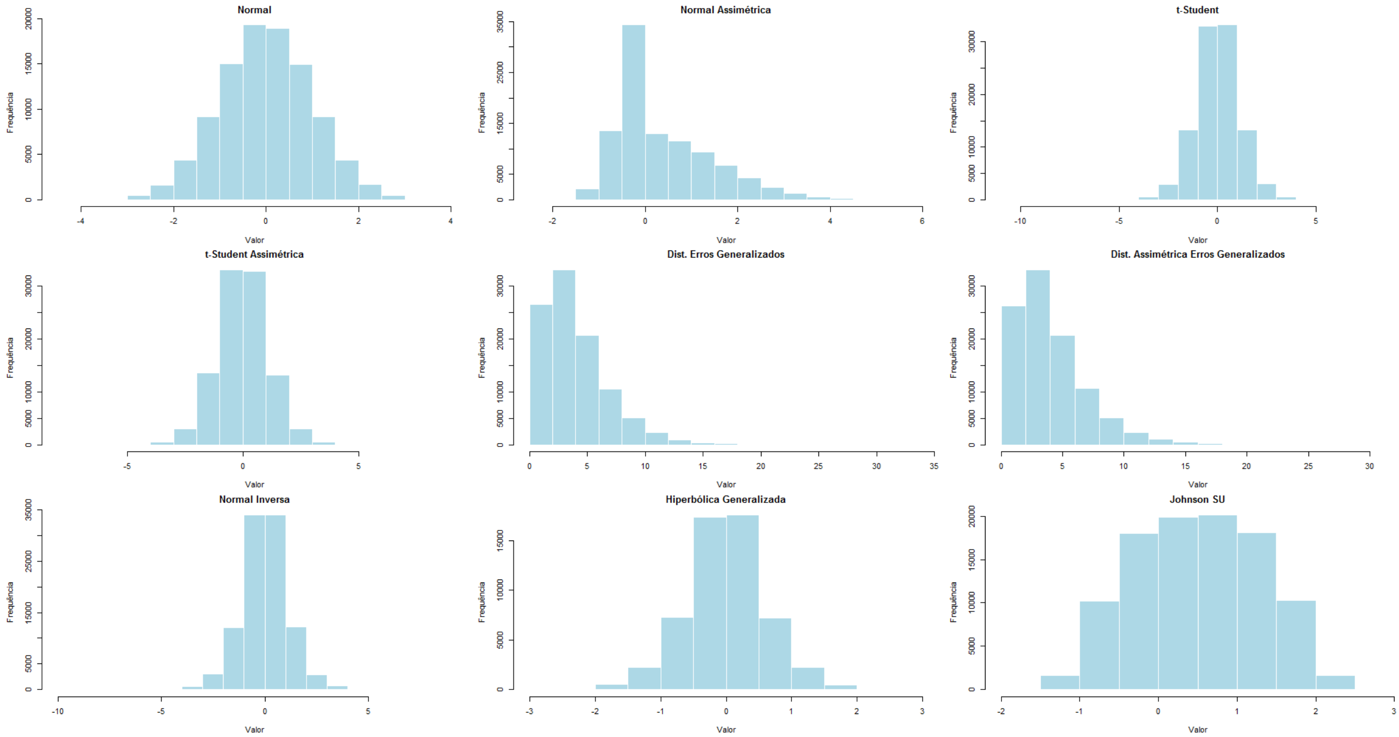
Variação GARCH	Equação
Exponential Generalized Autoregressive Conditional Heteroscedastic - eGARCH	$\ln \sigma_t^2 = \alpha_0 + \sum_{k=1}^{r} \alpha_k \frac{\lvert\varepsilon_{t-k}\rvert}{\sigma_{t-k}} + \sum_{k=1}^{r} \gamma_k \frac{\lvert\varepsilon_{t-k}\rvert}{\sigma_{t-k}} \epsilon_t + \sum_{l=1}^{m} \beta_l \ln \sigma_{t-l}^2$
Threshold Generalized Autoregressive Conditional Heteroscedastic - tGARCH	$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{r} \alpha_k \varepsilon_{t-k}^2 + \sum_{o=1}^{g} I_{t-o} \gamma_k \varepsilon_{t-o}^2 + \sum_{n=1}^{h} J_{t-n} \delta_n \sigma_{t-n}^2 \ \epsilon_t + \sum_{l=1}^{m} \beta_l \sigma_{t-l}^2$
Asymmetric Power Autoregressive Conditional Heteroscedastic - apARCH	$\sigma_t^\delta = \alpha_0 + \sum_{k=1}^{r} \alpha_k (\lvert\varepsilon_{t-k}\rvert - \gamma_k \varepsilon_{t-k})^\delta + \epsilon_t + \sum_{l=1}^{m} \beta_l \sigma_{t-l}^\delta$
Component Generalized Autoregressive Conditional Heteroscedastic - csGARCH	$\sigma_t^t = q_t + h_t$ Long-term component: $q_t = \omega_0 + p_0 q_{t-1} + \sum_{k=1}^{r} \phi_k (\varepsilon_{t-k}^2 - \sigma_{t-k}^2)$ Short-term component: $h_t = \beta_0 h_{t-1} + \sum_{l=1}^{m} \alpha_k (\varepsilon_{t-k}^2 - q_{t-k})$
Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedastic - gjrGARCH	$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{r} \alpha_k \varepsilon_{t-k}^2 + \sum_{o=1}^{g} I_{t-o} \gamma_o \varepsilon_{t-o}^2 + \epsilon_t + \sum_{l=1}^{m} \beta_l \sigma_{t-l}^2$
Non Linear Generalized Autoregressive Conditional Heteroscedastic - csGARCH	$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{r} \alpha_k f(\varepsilon_{t-k})^2 + \epsilon_t + \sum_{l=1}^{m} \beta_l \sigma_{t-l}^2$

Source: The autors

In general, both the gjrGARCh and tGARCH models incorporate a threshold component when modelling conditional volatility, allowing it to respond differently to positive and negative shocks. The eGARCH, nGARCH and apARCH models work with non-linear variations for volatility modelling. Finally, the csGARCH model divides conditional volatility into distinct components, each modelled separately. This provides greater flexibility in modelling different aspects of volatility, such as short-term and long-term volatility (Charles & Darné, 2019).

Furthermore, it is important to emphasise that the error ε_t must be assumed to follow a predefined distribution, originally defined as the normal distribution. However, previous studies, as described in section 2.3, identified a divergence between the empirical and theoretical distribution of errors, so other distributions that take into account asymmetry and kurtosis began to be used to estimate GARCH models. Figure 2 summarises some of the distributions commonly used.

Figure 2 - Histograms of Distributions Used in GARCH Models



Source: The autors

Using the free R software, ARMA(p,q)-GARCH(r,m) models were run for each of the 10 cryptocurrencies selected. To analyse the best modelling, three variations were considered:

Parameters p,q,r,m: the interval [0,2] was considered for each of these parameters, disregarding the configuration in which p=q=r=m=0 and in which r=m=0, which would indicate the absence of variance modelling in the model, thus totalling 36 possible configurations.

Distribution: the following distributions were considered: normal, asymmetric normal, Student's t, asymmetric Student's t, generalised error distribution, asymmetric generalised error distribution, inverse normal, Generalised Hyperbolic and Johnson's SU distribution.

GARCH variations: finally, in addition to traditional GARCH modelling, eGARCH, apARCH, csGARCH, tGARCH, gjrGARCH and nGARCH models were also run.

Thus, 2268 models were estimated for each currency. Furthermore, for each of these models, 80% of the sample was taken as the period for estimation and 20% for testing. Based on the MAPE (Mean Absolute Percentage Error) of the models, the best configurations for each cryptocurrency were identified. According to Equation 4, this indicator is calculated by the percentage sum of the absolute errors between the actual values X_i and the predicted values $\widehat{X_\iota}$.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{X_i-\widehat{X_\iota}}{X_i}\right| x100 \qquad (4)$$

When the best model was generated, it was used to calculate the VaR and ES for the time series. In order to avoid a fixed value for ES, it was decided to use a moving estimation window of 60 for this indicator.

3.3 Inclusion of External Variables

A final analysis is to compare the improvement of the GARCH modelling after including the FGI and MVRV as external variables. This inclusion aims to analyse whether these indicators of fear/greed and the relationship between the market value and the realised value of cryptocurrencies would have an impact on predicting the volatility of the series. Thus, the steps described in section 3.2 were repeated and, after calculating the VaR and ES, these were compared with those calculated previously.

4 ANALYSING THE RESULTS

4.1 Preliminary Analyses

From the data on the logarithmic returns of the cryptoassets, the average returns and annualised standard deviation were calculated, as well as the Sharpe Ratio (SR) and the historical VaR and ES. In addition, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were estimated to analyse the stationarity of the series. The results are shown in Table 2.

Table 2 - Basic Statistics of the Cryptocurrency Returns Series

	Average Annualised Returns	Annualised Standard Deviation	SR	VaR 5%	ES 5%	KPSS	ADF	PP
BTC	0.32462	0.56354	0.57604	-0.05232	-0.08406	0.671	-11.386**	-2022.475**
ETH	0.42889	0.71151	0.60279	-0.06746	-0.10793	0.927	-11.257**	-2047.322**
XRP	0.02969	0.86191	0.03445	-0.07416	-0.12236	0.052	-11.575**	-2001.426**
DOGE	0.70356	1.15071	0.61141	-0.07626	-0.1308	0.835	-11.378**	-1889.877**
BNB	0.54945	0.75726	0.72558	-0.0655	-0.10819	0.955	-10.288**	-2162.322**
ADA	0.25303	0.82096	0.30821	-0.07737	-0.11485	0.598	-10.713**	-2063.894**
TRX	0.23254	0.74215	0.31334	-0.06948	-0.11585	0.217	-12.049**	-2011.702**
BCH	0.01354	0.84904	0.01595	-0.07474	-0.12539	0.095	-12.275**	-2128.447**
LINK	0.46242	0.94777	0.48791	-0.08922	-0.13531	0.997	-12.224**	-2009.513**
LTC	-0.0207	0.76861	-0.02693	-0.0759	-0.11899	0.069	-12.135**	-1930.628**

Note: ***; **; * indicate significance at 1%, 5% and 10% respectively.

Source: The authors

Analysing the table provided reveals several important characteristics about the different cryptocurrencies in terms of returns, volatility, adjusted risk and the statistical properties of the time series. Firstly, when looking at the average annualised returns, DOGE stands out with the highest return (0.70356), followed by BNB (0.54945) and ETH (0.42889). In contrast, LTC shows a negative return (-0.0207), indicating an average annual loss.

In terms of volatility, DOGE again stands out, but this time negatively, with the highest annualised standard deviation (1.15071), signalling high volatility. BTC and BNB, on the other hand, show lower volatility with standard deviations of 0.56354 and 0.75726 respectively.

The IR, which measures risk-adjusted return, indicates that BNB has the best performance (0.72558), suggesting a good balance between return and risk. On the other hand, LTC has a negative Sharpe Ratio (-0.02693), suggesting that the associated risk does not compensate for the return.

Analysing the VaR at 5%, which estimates the maximum loss expected under normal market conditions for 5% of the worst-case scenarios, we see that LINK has the highest negative VaR (-0.08922), indicating greater potential losses. BTC has the lowest negative VaR (-0.05232), suggesting lower potential losses. The ES at 5%, which calculates the average expected loss in the 5% worst cases, shows that LINK also has the highest expected loss (-0.13531), indicating greater extreme risk. BTC again stands out positively with the lowest expected loss (-0.08406).

The KPSS, ADF and PP stationarity tests provide insight into the nature of the time series of cryptocurrency returns. It is important to emphasise that the KPSS establishes the null hypothesis that the series is stationary and therefore has no unit root, while the ADF and PP tests have the null hypothesis that the series has a unit root. It is therefore possible to corroborate that cryptocurrency returns are stationary series, which supports the use of ARMA-GARCH models.

4.2 ARIMA models and ARCH test

Continuing the analyses of the time series, the validity of applying models for heteroscedasticity of the variances of the time series was checked. To do this, the Lagrange Multiplier (LM) test was applied to ARCH modelling.

However, to carry out this procedure, it is first necessary to calculate the best ARMA model for each time series so that the LM test can then be applied, testing the null hypothesis of no correlation between the model's residuals. The best ARMA models were estimated for the 10 series of cryptocurrency returns using the auto.arima method in the forecast package, which identifies the values of p, d and q that minimise the AIC and SBIC criteria. Based on the best modelling, LM tests were carried out, the results of which are shown in Table 3. Furthermore, to enrich the analyses, Figure 3 shows, in addition to the graphs of cryptocurrency returns, the graphs of Autocorrelation Functions (ACF) and Partial Autocorrelation Functions (PACF).

Table 3 - LM ARCH Test's Results

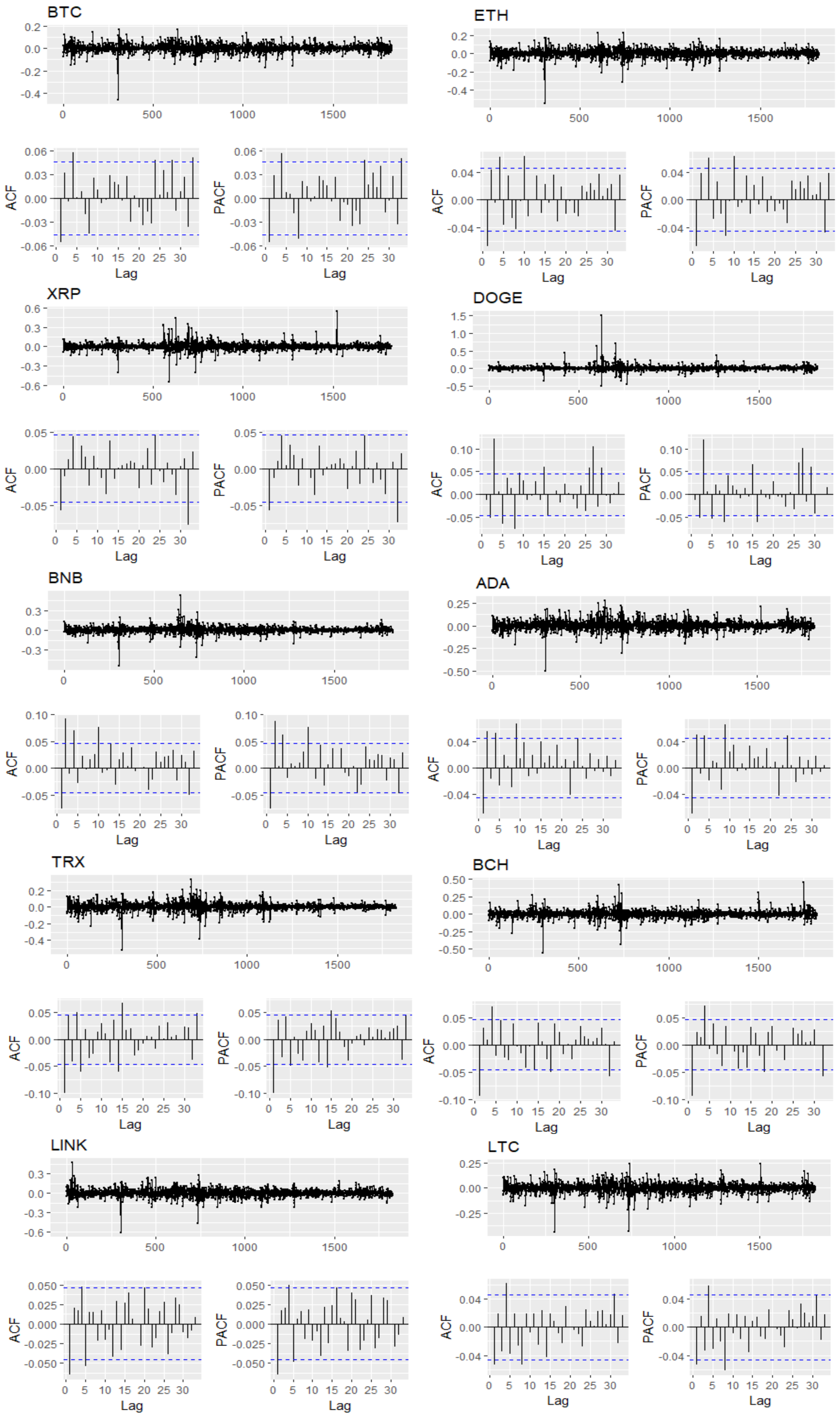
	Model	Order of the LM ARCH tests					
		4	8	12	16	20	24
BTC	ARIMA(2,0,2)	8503***	3238***	2116***	1575***	1237***	1026***
ETH	ARIMA(5,0,0)	6880***	2595***	1697***	1260***	983***	811***
XRP	ARIMA(2,0,3)	6541***	2974***	1962***	1389***	1086***	841***
DOGO	ARIMA(2,0,2)	22895***	10307***	6562***	4886***	3874***	3202***
BNB	ARIMA(2,0,2)	5760***	2254***	1190***	875***	674***	558***
ADA	ARIMA(2,0,2)	3414***	1391***	914***	659***	526***	437***
TRX	ARIMA(1,0,1)	6467***	2269***	1501***	1053***	839***	693***
BCH	ARIMA(5,0,4)	7421***	2816***	1865***	1225***	917***	751***
LINK	ARIMA(0,0,1)	5037***	1993***	1319***	953***	746***	617***
LTC	ARIMA(1,0,1)	4418***	1631***	1068***	785***	623***	515***

Note: ***; **; * indicate significance at 1%, 5% and 10% respectively.

Source: The authors

The results show that there is a correlation between the residuals of the ARIMA models that minimise the AIC and SBIC criteria. This corroborates the application of modelling for the volatility of the residuals, which is discussed in the next section.

Figure 3 - Graphs of cryptocurrency returns, ACF and PACF



Source: The authors

4.3 GARCH models

Based on the previous analyses, the validity of applying models to estimate the heteroscedasticity of the selected cryptocurrency series was verified. Thus, using the rugarch package, 2268 variations of ARMA-GARCH models were estimated, considering 80% of the total sample of return series. Forecasts were then made for the remaining 20% of the sample, selecting the configuration that minimised the MAPE for each asset. The results are shown in Table 4**.**

Table 4 - Results of the ARMA-GARCH Models for Cryptocurrencies

Cripto	Distribution	GARCH	Modelo	AIC	BIC	MSE	MAE	MAPE
BTC	Generalized Error Distribution	csGARCH	ARMA(0,2)GARCH(1,2)	-4.0330	-3.9968	0.0006	0.0168	0.9959%
ETH	Skew-Normal	GARCH	ARMA(1,1)GARCH(0,1)	-3.2388	-3.2171	0.0007	0.0181	0.9961%
XRP	Normal Inverse	GARCH	ARMA(2,0)GARCH(0,1)	-3.3896	-3.3643	0.0017	0.0222	0.9984%
DOGO	Normal Inverse	GARCH	ARMA(2,2)GARCH(1,0)	-3.3908	-3.3582	0.0018	0.0281	0.9944%
BNB	Normal	tGARCH	ARMA(2,1)GARCH(2,0)	-3.3626	-3.3301	0.0008	0.0181	0.9977%
ADA	Normal	apARCH	ARMA(1,1)GARCH(2,2)	-3.1096	-3.0698	0.0014	0.0261	0.9952%
TRX	Normal	GARCH	ARMA(2,2)GARCH(2,1)	-3.3750	-3.3424	0.0004	0.0135	0.9975%
BCH	Normal	tGARCH	ARMA(2,0)GARCH(2,2)	-3.1891	-3.1529	0.0027	0.0303	0.9984%
LINK	Skew-Student-T	tGARCH	ARMA(2,0)GARCH(2,0)	-2.9084	-2.8722	0.0016	0.0292	0.9960%
LTC	Normal	gjrGARCH	ARMA(2,2)GARCH(1,2)	-3.2325	-3.1963	0.0013	0.0234	0.9913%

Source: The authors

There are four interesting points about the results. Firstly, all the MAPEs of the models are less than 1%, thus highlighting that even limiting the values of the parameters p,q,r and m to between 0 and 2, the results, as well as being parsimonious, also have robust predictive power.

Furthermore, it should be noted that cryptocurrencies with higher annualised returns, such as ETH, DOGE, BNB and LINK, tend to use normal distributions or variations of the normal. In addition, for cryptocurrencies with greater risk, such as DOGE and LINK, tGARCH models are chosen to better capture extreme volatility. On the other hand, cryptocurrencies with a lower standard deviation, such as BTC and ETH, use GARCH models and less extreme distributions. These conclusions corroborate analyses such as those by Fung et al. (2022) and Acereda et al. (2020) and Ngunyi et al. (2019) on the importance of choosing the GARCH model used and the distribution assumed for the errors individually for each asset analysed.

4.3 Inclusion of External Regressors

In order to check whether the inclusion of two external variables, the FGI and the MVRV, would increase the predictive capacity of the volatility of cryptocurrency returns, new ARMA-GARCH models were estimated, and Table 5 shows the results of the models that minimised the MAPE.

Table 5 - Results of ARMA-GARCH Models with External Variables for Cryptocurrencies

Cripto	Distribution	GARCH	Modelo	AIC	BIC	MSE	MAE	MAPE
BTC	Johnson's SU	csGARCH	ARMA(0,0)GARCH(1,1)	-4.041637	-4.0055	0.0006	0.0168	0.9957%
ETH	Normal	eGARCH	ARMA(2,0)GARCH(1,2)	-3.353479	-3.3173	0.0007	0.0181	0.9966%
XRP	Normal	csGARCH	ARMA(2,1)GARCH(2,1)	-3.256535	-3.2131	0.0017	0.0222	0.9961%
DOGO	Normal	csGARCH	ARMA(2,2)GARCH(2,2)	-3.034956	-2.9843	0.0018	0.0281	0.9973%
BNB	Skew-Generalized Error Distribution	apARCH	ARMA(0,0)GARCH(1,2)	-3.6044	-3.5646	0.0008	0.0181	0.9993%
ADA	Normal	csGARCH	ARMA(2,2)GARCH(2,2)	-3.0350	-2.9843	0.0018	0.0281	0.9973%
TRX	Normal	csGARCH	ARMA(2,2)GARCH(2,2)	-3.0350	-2.9843	0.0018	0.0281	0.9973%
BCH	Generalized Error Distribution	nGARCH	ARMA(0,0)GARCH(2,2)	-3.4124	-3.3762	0.0027	0.0303	0.9988%
LINK	Normal Inverse	GARCH	ARMA(2,2)GARCH(2,1)	-2.9557	-2.9087	0.0016	0.0292	0.9920%
LTC	Generalized Hyperbolic	apARCH	ARMA(0,0)GARCH(2,2)	-3.380707	-3.3300	0.0013	0.0235	0.9979%

Source: The authors

From the data in Table 5, it can be seen that the inclusion of FGI and MVRV did not generate a uniform effect on the indicators of the ARMA-GARCH models used. With specific regard to MAPE, BTC, XRP, TRX and LINK showed a reduction in this index, reflecting improvements in forecasts, while the other cryptocurrencies showed an increase in these values. However, it is worth noting that all the variations obtained in the AMPE were less than 1%, signalling, a priori, a low forecasting gain when adding the external regressors.

Furthermore, it can be seen that the inclusion of external regressors in the modelling altered the parameters of the time series models. Thus, it can be concluded that the inclusion of external variables can alter the dynamics of cryptocurrency returns, thus reflecting new patterns of error variance, in turn requiring more flexible distributions to be properly captured. The change in distributions indicates that external variables have a significant impact on the way returns are distributed.

In summary, the inclusion of external variables in the ARMA-GARCH models resulted in adjustments to the models and distributions, reflecting occasional improvements in the prediction of some cryptocurrencies and minimal variation in overall predictive performance. The models chosen indicate a preference for distributions and structures that better capture the specific nature of the volatility of each cryptocurrency, with emphasis on the use of more robust models such as csGARCH and tGARCH for cryptocurrencies with higher volatility.

4.4 Analysing VaR and ES

As discussed above, the inclusion of external regressors affects the choice of time series model parameters that minimise the MAPE of forecasts. In this sense, it is interesting to comparatively analyse the volatility estimation performance of models controlling for the inclusion of external regressors. In this respect, Table 6 shows the VaR and ES results calculated for the cryptocurrency series based on the volatility estimates generated by the ARMA-GARCH models according to the parameters highlighted in Tables 4 and 5.

Table 6 - VaR and ES Results Estimated from ARMA-GARCH Models

Criptocurrency	Result	VaR	ES	VaR_Ext	ES_Ext
BTC	Success	95.87%	98.13%	96.66%	98.47%
	Failure	4.13%	1.87%	3.34%	1.53%
ETH	Success	95.36%	98.59%	97.29%	98.76%
	Failure	4.64%	1.41%	2.71%	1.24%
XRP	Success	96.44%	98.64%	97.91%	98.87%
	Failure	3.56%	1.36%	2.09%	1.13%
DOGO	Success	96.72%	97.85%	98.02%	97.85%
	Failure	3.28%	2.15%	1.98%	2.15%
BNB	Success	97.40%	98.76%	95.70%	97.91%
	Failure	2.60%	1.24%	4.30%	2.09%
ADA	Success	97.45%	98.64%	97.62%	98.42%
	Failure	2.55%	1.36%	2.38%	1.58%
TRX	Success	97.45%	97.85%	97.23%	97.96%
	Failure	2.55%	2.15%	2.77%	2.04%
BCH	Success	97.34%	98.53%	95.81%	98.02%
	Failure	2.66%	1.47%	4.19%	1.98%
LINK	Success	95.87%	97.96%	95.81%	97.96%
	Failure	4.13%	2.04%	4.19%	2.04%
LTC	Success	96.55%	97.79%	95.08%	97.68%
	Failure	3.45%	2.21%	4.92%	2.32%

Note: ***; **; * indicate significance at 1%, 5% and 10% respectively.

Source: The authors

Table 6 was constructed based on the analysis between the predicted VaR and ES values and the actual return of each cryptocurrency. Thus, “Success” indicates the percentage of times the actual return was higher than the VaR/ES in the data sample, and “Failure” indicates the percentage of times the actual return showed a negative result that exceeded these risk indices.

Overall, the calculated VaRs had a “Success” rate of 96.65% in estimating risk, implying a global failure margin of 3.35%. For the ES, these values are 98.27% and 1.73%, respectively, indicating that, in general, the ES shows a lower margin of error for worse outcomes when compared to VaR, as expected by the literature (Acerbi & Tasche, 2002; Acereda et al., 2020; Malek et al., 2023). Thus, it is corroborated that the ES is a more adequate tool for risk management for cryptocurrency investors, aligning with the conclusions of Huang et al. (2024).

To analyze the efficiency gain of risk management considering external variables (VaR_Ext and ES_EXT), it is observed that, overall, the models have a success rate of 96.71%, indicating a performance gain of 0.07% regarding the VaR methodology. However, regarding the ES, it had a success rate of 98.19%, implying a loss of efficiency of about 0.09%. Therefore, it is noticed that, although VaR presents a benefit when estimated considering external regressors, the ES does not share this effect.

To analyze the significance of the differences in average success rates of VaR and ES when considering the effect of external variables, Table 7 was generated. It shows the variation in success rates and the respective level of significance of this variation.

Table 7 – Difference in VaR and ES When Considering External Regressors

Criptocurrency	VaR	ES
BTC	0.79%**	1.81%**
ETH	1.92%***	0.17%
XRP	1.47%***	0.17%
DOGO	1.3%***	0.00%
BNB	-1.7%***	-0.85%***
ADA	0.17%	-0.23%***
TRX	-0.23%	0.11%
BCH	-1.53%***	-0.51%***
LINK	-0.06%	0.00%
LTC	-1.47%***	-0.11%

Note: ***; **; * indicate significance at 1%, 5% and 10% respectively.

Source: The authors

According to the data, BTC, ETH, XRP, and DOGE show significant gains when using VaR_Ext compared to VaR. Conversely, BNB, BCH, and LTC show a loss of this efficiency.

Therefore, FGI and MVRV appear more useful for predicting the volatility of the four cryptocurrencies with the largest market share, whereas for others, their effects do not seem relevant or even worsen the quality of predictions.

Regarding ES_Ext, it was significantly superior to ES only for BTC, while for BNB, ADA, and BCH, the results indicate a loss of efficiency. Thus, it is concluded that ES, although better than VaR for risk management, does not benefit from using FGI and MVRV within the context of ARMA-GARCH models, except in the case of BTC.

5 FINAL CONSIDERATIONS

Investor interest in the cryptocurrency market has been growing in recent years. However, the higher degree of volatility in cryptocurrencies also necessitates greater risk management when including such assets in portfolios. Aligned with this concern, the present study focused on analyzing the capability of using ARMA-GARCH models for risk management of the top 10 cryptocurrencies, excluding stablecoins, with over 5 years in the market.

To achieve this, 2268 time series model estimations were conducted for these assets, aiming to identify model parameter variations that minimized the mean of mean absolute errors. As a result, the MAPEs obtained for the best identified models were less than 1%, thus reflecting the quality of predictions. Using these more efficient models, VaR and ES were estimated for the selected cryptocurrencies.

Overall, the success rate of both metrics exceeded 95%, thereby reflecting the suitability of the proposed methodology for risk management. However, it is worth noting that ES proved to be a more accurate alternative than VaR.

Subsequently, the analyses of the models were revisited considering two external variables, FGI and MVRV. Comparative analyses of VaR_Ext revealed that the four largest cryptocurrencies (BTC, ETH, XRP, and DOGE) significantly improved performance by including these variables in ARMA-GARCH modeling. However, for ES_Ext, it proved more efficient only for BTC, while for 3 out of the other 9 cryptocurrencies, it was statistically less efficient than traditional ES.

Another important result of this study was the lack of clear identification of a pattern in model parameter selection, especially in variations of the GARCH family and error distributions. Thus, it reinforces the need to estimate specific models for each cryptocurrency, aiming to identify the one that adjusts most efficiently to risk heterogeneity, kurtosis, and skewness.

Limitations of this study include three points. Firstly, cryptocurrency returns were calculated based on USD quotes. Additionally, only two external variables were considered, while other relevant effects could be included in ARMA-GARCH modeling adjustments. Finally, this study focused on cryptocurrencies with the highest market capitalization, selecting 10 out of over 9,000 coins.

Therefore, for future research, it is suggested to first replicate this study considering the exchange rate effect of the dollar to expand the study's conclusions. Furthermore, it is also suggested to consider quotations in terms of BTC rather than USD, as this change in scale may alter the volatility of time series data.

Secondly, expanding the scope of external variables used aims to further enhance the predictive capability of models and the efficiency of estimated VaR and ES. Finally, it is suggested to study risk management comparatively between cryptocurrencies with higher and lower market participation, aiming to identify changes in the conclusions of this study when analyzing smaller cryptocurrencies.

REFERENCES

Abreu, D. P. A. de, Coaguila, R. A. I., & Camargos, M. A. de. (2022). Evolution of the degree of efficiency of the cryptocurrency market from 2014 to 2020: An analysis based on its fractal components. *Revista de Administração Da UFSM*, *15*(2), Artigo 2.

Acerbi, C., & Tasche, D. (2002). On the coherence of expected shortfall. *Journal of Banking & Finance*, *26*(7), 1487–1503. https://doi.org/10.1016/S0378-4266(02)00283-2

Acereda, B., Leon, A., & Mora, J. (2020). Estimating the expected shortfall of cryptocurrencies: An evaluation based on backtesting. *Finance Research Letters*, *33*, 101181. https://doi.org/10.1016/j.frl.2019.04.037

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). Coherent Measures of Risk. *Mathematical Finance*, *9*(3), 203–228. https://doi.org/10.1111/1467-9965.00068

Baur, D. G., & Dimpfl, T. (2018). Asymmetric volatility in cryptocurrencies. *Economics Letters*, *173*, 148–151. https://doi.org/10.1016/j.econlet.2018.10.008

Baur, D. G., Hong, K., & Lee, A. D. (2018). Bitcoin: Medium of exchange or speculative assets? *Journal of International Financial Markets, Institutions and Money*, *54*, 177–189. https://doi.org/10.1016/j.intfin.2017.12.004

Cai, C. W., Xue, R., & Zhou, B. (2023). Cryptocurrency puzzles: A comprehensive review and re-introduction. *Journal of Accounting Literature*, *46*(1), 26–50. https://doi.org/10.1108/JAL-02-2023-0023

Charfeddine, L., Benlagha, N., & Maouchi, Y. (2020). Investigating the dynamic relationship between cryptocurrencies and conventional assets: Implications for financial investors. *Economic Modelling*, *85*, 198–217. https://doi.org/10.1016/j.econmod.2019.05.016

Charles, A., & Darné, O. (2019). The accuracy of asymmetric GARCH model estimation. *International Economics*, *157*, 179–202. https://doi.org/10.1016/j.inteco.2018.11.001

Conlon, T., & McGee, R. (2020). Safe haven or risky hazard? Bitcoin during the Covid-19 bear market. *Finance Research Letters*, *35*, 101607. https://doi.org/10.1016/j.frl.2020.101607

Engle, R. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Business & Economic Statistics*, *20*(3), 339–350. https://doi.org/10.1198/073500102288618487

Fung, K., Jeong, J., & Pereira, J. (2022). More to cryptos than bitcoin: A GARCH modelling of heterogeneous cryptocurrencies. *Finance Research Letters*, *47*, 102544. https://doi.org/10.1016/j.frl.2021.102544

Gandal, N., & Halaburda, H. (2014). *Competition in the Cryptocurrency Market* (SSRN Scholarly Paper 2501640). https://papers.ssrn.com/abstract=2501640

Hasan, F., Al-Okaily, M., Choudhury, T., & Kayani, U. (2024). A comparative analysis between FinTech and traditional stock markets: Using Russia and Ukraine war data. *Electronic Commerce Research*, *24*(1), 629–654. https://doi.org/10.1007/s10660-023-09734-0

Huang, Y., Wang, H., Chen, Z., Feng, C., Zhu, K., Yang, X., & Yang, W. (2024). Evaluating Cryptocurrency Market Risk on the Blockchain: An Empirical Study Using the ARMA-GARCH-VaR Model. *IEEE Open Journal of the Computer Society*, *5*, 83–94. https://doi.org/10.1109/OJCS.2024.3370603

Kristoufek, L., & Vosvrda, M. (2019). Cryptocurrencies market efficiency ranking: Not so straightforward. *Physica A: Statistical Mechanics and Its Applications*, *531*, 120853. https://doi.org/10.1016/j.physa.2019.04.089

Lánský, J. (2017). Bitcoin System. *Acta Informatica Pragensia*, *6*(1), 20–31. https://doi.org/10.18267/j.aip.97

Malek, J., Nguyen, D. K., Sensoy, A., & Tran, Q. V. (2023). Modeling dynamic VaR and CVaR of cryptocurrency returns with alpha-stable innovations. *Finance Research Letters*, *55*, 103817. https://doi.org/10.1016/j.frl.2023.103817

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*(1), 77–91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

Nakamoto, S. (2008). *Bitcoin: A Peer-to-Peer Electronic Cash System*. https://bitcoin.org/bitcoin.pdf

Ngunyi, A., Mundia, S., & Omari, C. (2019). *Modelling Volatility Dynamics of Cryptocurrencies Using GARCH Models*. http://repository.dkut.ac.ke:8080/xmlui/handle/123456789/978

Suryono, R. R., Budi, I., & Purwandari, B. (2020). Challenges and Trends of Financial Technology (Fintech): A Systematic Literature Review. *Information*, *11*(12), Artigo 12. https://doi.org/10.3390/info11120590

Trimborn, S., & Härdle, W. K. (2018). CRIX an Index for cryptocurrencies. *Journal of Empirical Finance*, *49*, 107–122. https://doi.org/10.1016/j.jempfin.2018.08.004

Troster, V., Tiwari, A. K., Shahbaz, M., & Macedo, D. N. (2019). Bitcoin returns and risk: A general GARCH and GAS analysis. *Finance Research Letters*, *30*, 187–193. https://doi.org/10.1016/j.frl.2018.09.014

Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters*, *148*, 80–82. https://doi.org/10.1016/j.econlet.2016.09.019

Wang, G.-J., Ma, X., & Wu, H. (2020). Are stablecoins truly diversifiers, hedges, or safe havens against traditional cryptocurrencies as their name suggests? *Research in International Business and Finance*, *54*, 101225. https://doi.org/10.1016/j.ribaf.2020.101225